Mathematical modeling and definition of surface strengthening best temperature conditions

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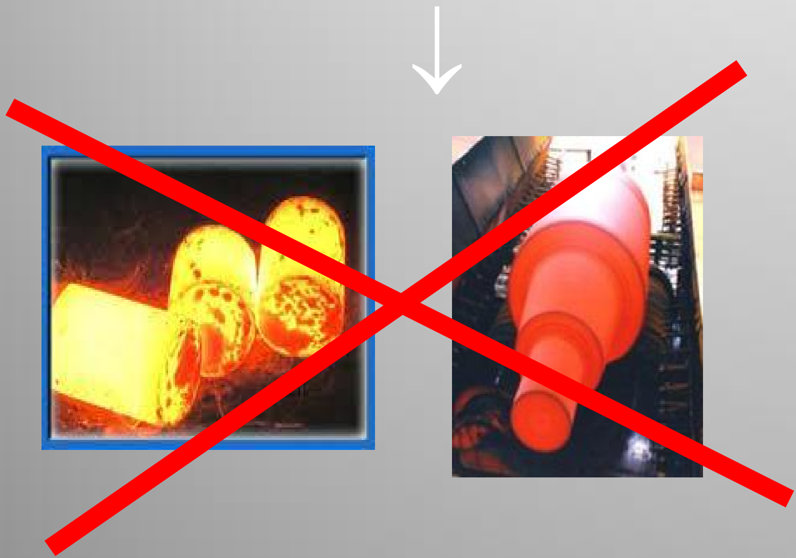
Mechanical engineering

Rocket engineering

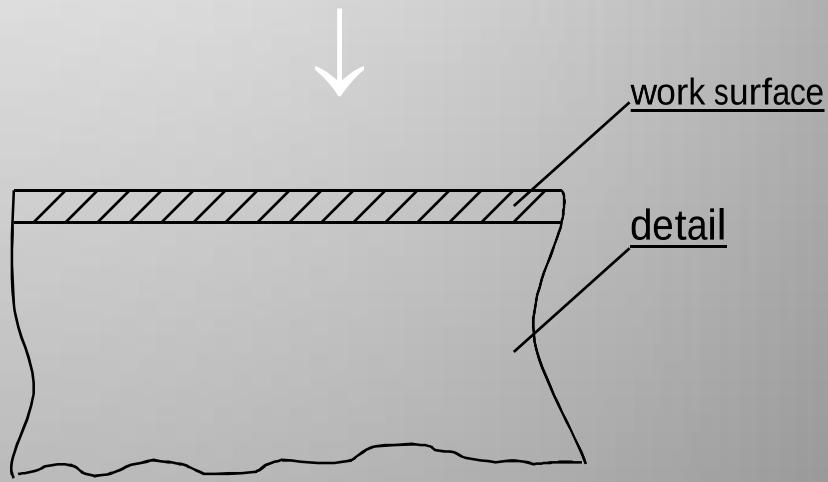
Space technologies

In many cases we can see that whole cross section is influenced the thermal treatment

Typical method of treatment



Laser treatment



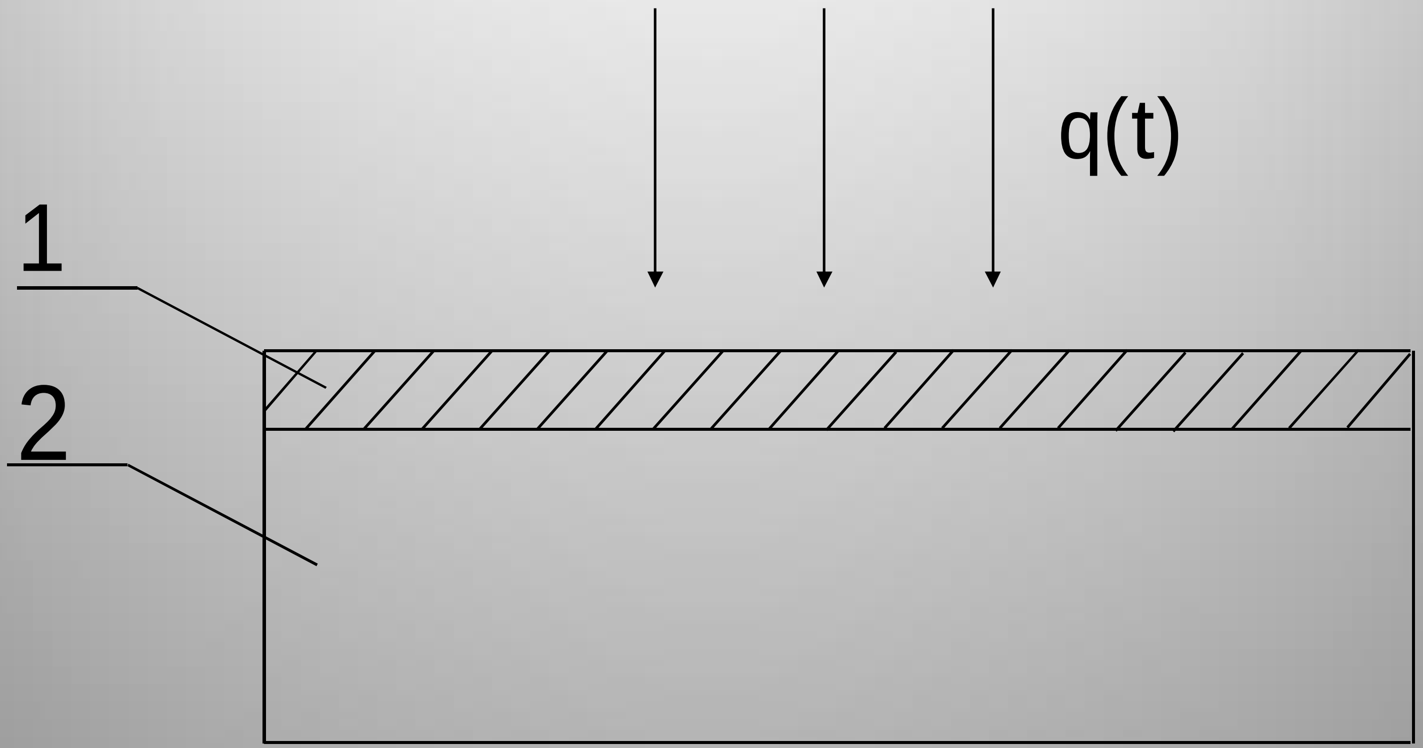
Research goal

to build mathematical model, by using which we will be able to obtain the optimal function of the laser treatment for strengthening surface.

The first phase of my research work was the obtaining of the solution for the following problem:

"Mathematic modeling of thermal processes within processing coating by the concentrated energy flow with melting and evaporation consideration"

This problems can be represent the following way:



where 1- coating of our system (Titan), 2 – base of our system (Fe), q(t) – function of the laser treatment

In process of solving this problem the solution was split into 2 stages

1) the stage of heating system up till reaching melting temperature on the surface;
2) the stage of heating up with the process of surface melting when it is also important to consider surface evaporation as quite an energy-consuming process

The Mathematic formulation of the problem of the first stage

$$c_1(T_1)\rho_1(T_1)\frac{\partial T_1}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_1(T_1)\frac{\partial T_1}{\partial x}\right), \quad 0 < x < l_1;$$

$$\mathrm{c}_2(\mathrm{T}_2)\rho_2(\mathrm{T}_2)\frac{\partial \mathrm{T}_2}{\partial \mathrm{t}} = \frac{\partial}{\partial \mathrm{x}}\left(\lambda_2(\mathrm{T}_2)\frac{\partial \mathrm{T}_2}{\partial \mathrm{x}}\right), \quad \mathrm{l}_1 < \mathrm{x} < \mathrm{l}_2,$$

Initial and boundary conditions:

$$T_i(x,0) = T_0; \quad i = 1,2$$

$$m_1\lambda_1(T_1)\frac{\partial T_1}{\partial x}\bigg|_{x=0} + f_1(t)T_1\big|_{x=0} = q_1(t)$$

$$m_2\lambda_2(T_2)\frac{\partial T_2}{\partial x}\bigg|_{x=l_2} + f_2(t)T_2\big|_{x=l_2} = q_2(t)$$

$$T_1\big|_{x=l_1} = T_2\big|_{x=l_1}; \qquad \lambda_1(T_1)\frac{\partial T_1}{\partial x}\bigg|_{x=l_1} = \lambda_2(T_2)\frac{\partial T_2}{\partial x}\bigg|_{x=l_1};$$

Mathematic problem formulation of the second stage

$$\rho_1(T_1)\left(c_1(T_1)+L_1\delta(T_1-T_{n,1})\right)\frac{\partial T_1}{\partial t}=\frac{\partial}{\partial x}\left(\lambda_1(T_1)\frac{\partial T_1}{\partial x}\right), y_0(t)<x<l_1$$

$$c_2(T_2)\rho_2(T_2)\frac{\partial T_2}{\partial t}=\frac{\partial}{\partial x}\left(\lambda_2(T_2)\frac{\partial T_2}{\partial x}\right), \qquad l_1<x<l_2,$$

Initial and boundary conditions:

$$T_i(x,t')=T_{0,i}(x), \quad i=1,2;$$

$$\lambda_1(T_1)\frac{\partial T_1}{\partial x}\bigg|_{x=y_0(t)}=f_1(t)T_1\big|_{x=y_0(t)}-q_1(t)+\rho_1\left(T_1\big|_{x=y_0(t)}\right)Q_1\frac{dy_0}{dt};$$

where $\frac{dy_0}{dt}=v_*\exp\left\{-T^*/T(y_0(t),t)\right\}; \qquad y_0\big|_{t=t'}=0.$

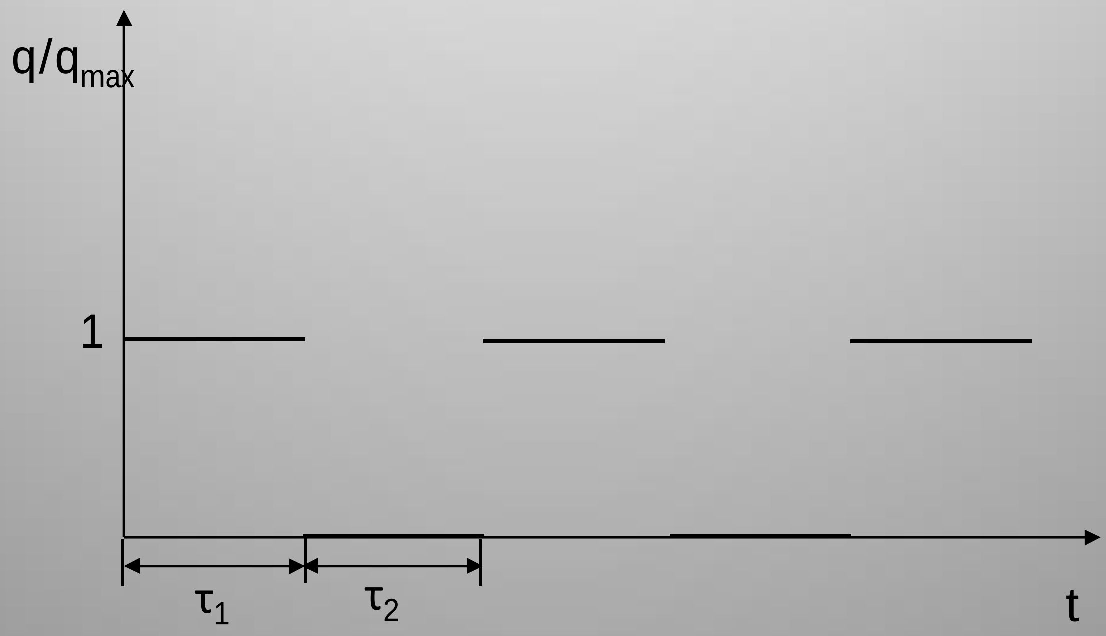
$$m_2\lambda_2(T_2)\frac{\partial T_2}{\partial x}\bigg|_{x=l_2}+f_2(t)T_2\big|_{x=l_2}=q_2(t),$$

$$T_1\big|_{x=l_1}=T_2\big|_{x=l_1}; \qquad \lambda_1(T_1)\frac{\partial T_1}{\partial x}\bigg|_{x=l_1}=\lambda_2(T_2)\frac{\partial T_2}{\partial x}\bigg|_{x=l_1}.$$

In process of solving there were used the following types of laser treatments

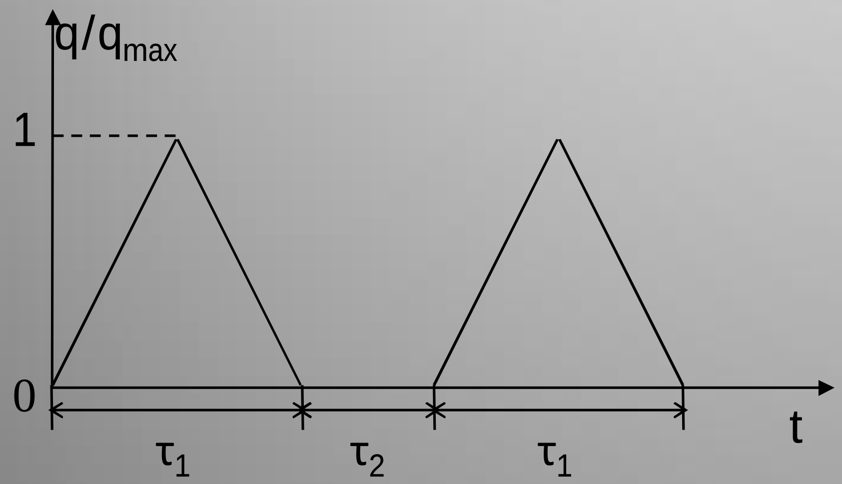
Pulse of “banch” type

$$q(t) = \begin{cases} q_{\max}; & i\tau < t \le i \cdot \tau + \tau_1, \\ 0, i\tau + \tau_1 < t \le (i+1)\tau. \end{cases}$$



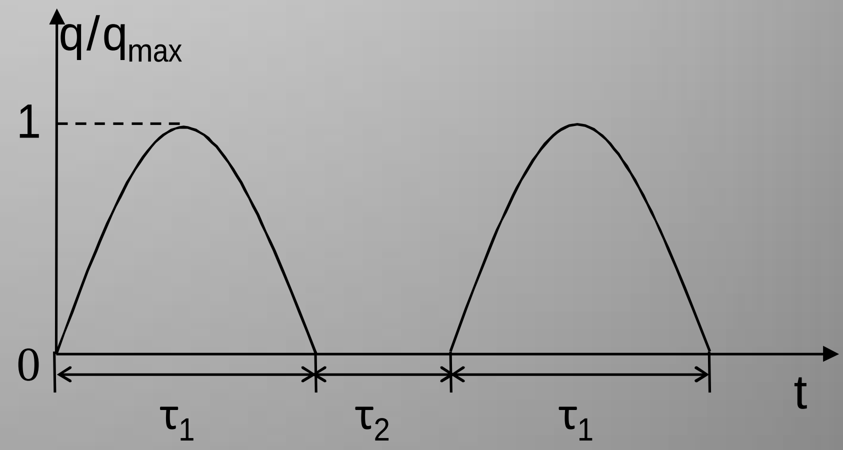
Pulse of a triangle form

$$q(t)=\begin{cases} q_{\max}\dfrac{t}{\tau_1}, & i\cdot\tau < t \le i\cdot\tau+\dfrac{\tau_1}{2}; \\ q_{\max}\left(1-\dfrac{t}{\tau_1}\right), & i\cdot\tau+\dfrac{\tau_1}{2} \le t \le i*\tau+\tau_1, \\ 0, i\cdot\tau+\tau_1 \le t \le (i+1)\tau \end{cases}$$



Sinusoidal pulse

$$q(t)=\begin{cases} q_{\max}\sin\left(\dfrac{\pi}{\tau_1}(t-i\tau)\right); & i\tau < t \le i\tau+\tau_1, \\ 0, i\tau+\tau_1 < t \le (i+1)\tau. \end{cases}$$



Method of solving the problem for the first stage

$$c_{1,k-1}^{j-1}\rho_{1,k-1}^{j-1}\frac{T_{1,k-1}^{j}-T_{1,k-1}^{j-1}}{\tau}=\frac{2}{h+h_k}\left[\lambda_{1,(k-1)+\frac{1}{2}}^{j-1}\frac{T_{1,k}^{j}-T_{1,k-1}^{j}}{h_k}-\lambda_{1,(k-2)+\frac{1}{2}}^{j-1}\frac{T_{1,k-1}^{j}-T_{1,k-2}^{j}}{h_k}\right],$$

$$c_{2,k+1}^{j-1}\rho_{2,k+1}^{j-1}\frac{T_{2,k+1}^{j}-T_{2,k+1}^{j-1}}{\tau}=\frac{2}{h_{k+1}+h}\left[\lambda_{2,(k+1)+\frac{1}{2}}^{j-1}\frac{T_{2,k+2}^{j}-T_{2,k+1}^{j}}{h}-\lambda_{2,k+\frac{1}{2}}^{j-1}\frac{T_{2,k+1}^{j}-T_{2,k+2}^{j}}{h_{k+1}}\right],$$

Difference analogues of external boundary conditions:

$$m_1\lambda_{1,0}^{j-1}\frac{T_{1,1}^{j}-T_{1,0}^{j}}{h}+f_1^{\,j}T_{1,0}^{j}=g_1^{j},\quad m_2\lambda_{2,N'}^{j-1}\frac{T_{2,N'}^{j}-T_{2,N'-1}^{j}}{h}+f_2^{\,j}T_{2,N'}^{j}=g_2^{j},$$

Boundary conditions on the contact surface:

$$T_{1,k}^{j}=T_{2,k}^{j},\qquad \lambda_{1,k}^{j-1}\frac{T_{1,k}^{j}-T_{1,k-1}^{j}}{h_k}=\lambda_{2,k}^{j-1}\frac{T_{2,k+1}^{j}-T_{2,k}^{j}}{h_{k+1}},$$

Method of solving the problem for the first stage

In internal nodes are not adjacent to x_k approximation will be as follows

for $i = 1,\dots,k-2,$

$$c_{1,i}^{j-1}\rho_{1,i}^{j-1}\frac{T_{1,i}^{j}-T_{1,i}^{j-1}}{\tau}=\frac{1}{h}\left[\lambda_{1,i+\frac{1}{2}}^{j-1}\frac{T_{1,i+1}^{j}-T_{1,i}^{j}}{h}-\lambda_{1,(i-1)+\frac{1}{2}}^{j-1}\frac{T_{1,i}^{j}-T_{1,i-1}^{j}}{h}\right],$$

for $i = k+2,\dots,N'-1,$

$$c_{2,i}^{j-1}\rho_{2,i}^{j-1}\frac{T_{2,i}^{j}-T_{2,i}^{j-1}}{\tau}=\frac{1}{h}\left[\lambda_{2,i+\frac{1}{2}}^{j-1}\frac{T_{2,i+1}^{j}-T_{2,i}^{j}}{h}-\lambda_{2,(i-1)+\frac{1}{2}}^{j-1}\frac{T_{2,i}^{j}-T_{2,i-1}^{j}}{h}\right],$$

Method of solving the problem for the second stage

In order to get to the difference scheme we should replace the delta function with smeared delta function at the interval ($T_{n,1}$ - Δ, $T_{n,1}$ + Δ)

$$\rho_1 \tilde{c}_1(T_1) = \rho_1(T)\left(c_1(T_1) + L_1 \delta(T_1 - T_{n,1^*}, \Delta)\right)$$

where $\rho_1 \tilde{c}_1(T_1) = c_{1,\text{solid}}(T_1)\rho_{1,\text{solid}}(T_1)$ when $T_1 < T_{n,1} - \Delta,$

$\rho_1 \tilde{c}_1(T_1) = c_{1,\text{liquid}}(T_1)\rho_{1,\text{liquid}}(T_1)$ when $T_1 > T_{n,1} + \Delta,$

and when ($T_{n,1}$ - Δ, $T_{n,1}$ + Δ)

$$\int_{T_{n,1}-\Delta}^{T_{n,1}+\Delta} \rho_1 \tilde{c}_1(T_1)\,dT_1 = \rho_1 L_1 + \int_{T_{n,1}-\Delta}^{T_{n,1}} \rho_{1,\text{т}}(T_1) c_{1,\text{т}}(T_1)\,dT_1 + \int_{T_{n,1}}^{T_{n,1}+\Delta} \rho_{1,\text{ж}}(T_1) c_{1,\text{ж}}(T_1)\,dT_1,$$

Method of solving the problem for the second stage

In order to get to the difference scheme we should replace the delta function with smeared delta function at the interval ($T_{n,1} - \Delta$, $T_{n,1} + \Delta$)

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$\rho_1 \tilde{c}_1(T_1) = c_{1,\mathrm{liquid}}(T_1)\rho_{1,\mathrm{liquid}}(T_1)$ when $T_1 > T_{n,1} + \Delta,$

and when ($T_{n,1} - \Delta$, $T_{n,1} + \Delta$)

$$\int_{T_{n,1}-\Delta}^{T_{n,1}+\Delta} \rho_1 \tilde{c}_1(T_1) dT_1 = \rho_1 L_1 + \int_{T_{n,1}-\Delta}^{T_{n,1}} \rho_{1,\text{т}}(T_1) c_{1,\text{т}}(T_1) dT_1 + \int_{T_{n,1}}^{T_{n,1}+\Delta} \rho_{1,\text{ж}}(T_1) c_{1,\text{ж}}(T_1) dT_1,$$

$$\rho_1(T_1)\left(c_1(T_1) + L_1 \delta(T_1 - T_{n,1})\right)\frac{\partial T_1}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_1(T_1)\frac{\partial T_1}{\partial x}\right) >>>>$$

$$>>>> \tilde{\rho}_1 \tilde{c}_1(T_1)\frac{\partial T_1}{\partial t} = \frac{\partial}{\partial x}\left(\tilde{\lambda}_1(T_1)\frac{\partial T_1}{\partial x}\right)$$

Method of solving the problem for the second stage

In adjacent nodes with x_k (nodes x_{k-1}, x_{k+1}) as well as in the node adjacent with the evaporation front $x_{r_i^j+1}$ the heat conduction equation is approximated as follows:

$$\rho_1 \tilde{c}^{\,j-1}_{1,r_0^j+1} \frac{T^{\,j}_{1,r_0^j+1} - T^{\,j-1}_{1,r_0^j+1}}{\tau} = \frac{2}{h_{r_0^j} + h}\left[\tilde{\lambda}^{\,j-1}_{1,(r_0^j-1)+\frac{1}{2}} \frac{T^{\,j}_{1,r_0^j+2} - T^{\,j}_{1,r_0^j+1}}{h_{r_0^j}} - \tilde{\lambda}^{\,j-1}_{1,(r_0^j-2)+\frac{1}{2}} \frac{T^{\,j}_{1,r_0^j+1} - T^{\,j}_{1,r_0^j}}{h_{r_0^j}}\right],$$

$$\rho \tilde{c}^{\,j-1}_{1,k-1} \frac{T^{\,j}_{1,k-1} - T^{\,j-1}_{1,k-1}}{\tau} = \frac{2}{h + h_k}\left[\tilde{\lambda}^{\,j-1}_{1,(k-1)+\frac{1}{2}} \frac{T^{\,j}_{1,k} - T^{\,j}_{1,k-1}}{h_k} - \tilde{\lambda}^{\,j-1}_{1,(k-2)+\frac{1}{2}} \frac{T^{\,j}_{1,k-1} - T^{\,j}_{1,k-2}}{h}\right]$$

$$c^{\,j-1}_{2,k+1} \rho^{\,j-1}_{2,k+1} \frac{T^{\,j}_{2,k+1} - T^{\,j-1}_{2,k+1}}{\tau} = \frac{2}{h_{k+1} + h}\left[\lambda^{\,j-1}_{2,(k+1)+\frac{1}{2}} \frac{T^{\,j}_{2,k+2} - T^{\,j}_{2,k+1}}{h} - \lambda^{\,j-1}_{2,k+\frac{1}{2}} \frac{T^{\,j}_{2,k+1} - T^{\,j}_{2,k}}{h_{k+1}}\right]$$

Method of solving the problem for the second stage

for $i = r_0^j + 2,...,k-2,$

$$c_{2,i}^{j-1}\rho_{2,i}^{j-1}\frac{T_{2,i}^{j}-T_{2,i}^{j-1}}{\tau}=\frac{1}{h}\left[\lambda_{2,i+\frac{1}{2}}^{j-1}\frac{T_{2,i+1}^{j}-T_{2,i}^{j}}{h}-\lambda_{2,(i-1)+\frac{1}{2}}^{j-1}\frac{T_{2,i}^{j}-T_{2,i-1}^{j}}{h}\right],$$

for $i = k+2,...,N'-1,$

$$c_{2,i}^{j-1}\rho_{2,i}^{j-1}\frac{T_{2,i}^{j}-T_{2,i}^{j-1}}{\tau}=\frac{1}{h}\left[\lambda_{2,i+\frac{1}{2}}^{j-1}\frac{T_{2,i+1}^{j}-T_{2,i}^{j}}{h}-\lambda_{2,(i-1)+\frac{1}{2}}^{j-1}\frac{T_{2,i}^{j}-T_{2,i-1}^{j}}{h}\right],$$

where $r_0^j = \left[\frac{y_0^j}{h}\right]$ y_0^j - the coordinate of the front of surface evaporation

Method of solving the problem for the second stage

difference analogue of boundary conditions at the external surface

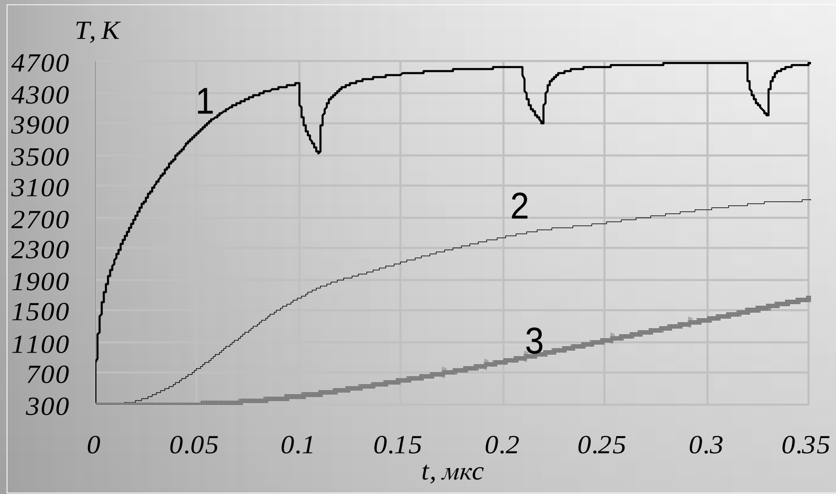
$$\tilde{\lambda}^{j-1}_{1,r_0^j,}\frac{T^j_{1,r_0^j+1}-T^j_{1,r_0^j,}}{h_{r_0^j}} = f_1^{\,j} T^j_{1,k_0^j,} - q_1^j - \rho^{j-1}_{1,r_0^j}\cdot Q_1\cdot\frac{y_0^{\,j}-y_0^{\,j-1}}{\tau},$$

$$m_2\lambda^{j-1}_{2,N'}\frac{T^j_{2,N'}-T^j_{2,N'-1}}{h} + f_2^{\,j} T^j_{2,N'} = q_2^j,$$

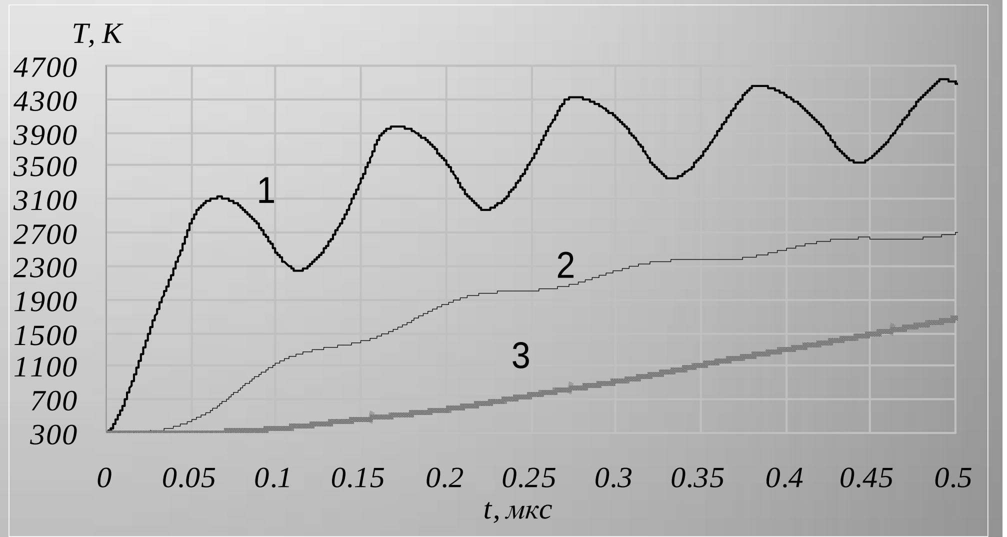
on the contact surface

$$T^j_{1,k} = T^j_{2,k}, \qquad \lambda^{j-1}_{1,k}\frac{T^j_{1,k}-T^j_{1,k-1}}{h_k} = \lambda^{j-1}_{2,k}\frac{T^j_{2,k+1}-T^j_{2,k}}{h_{k+1}},$$

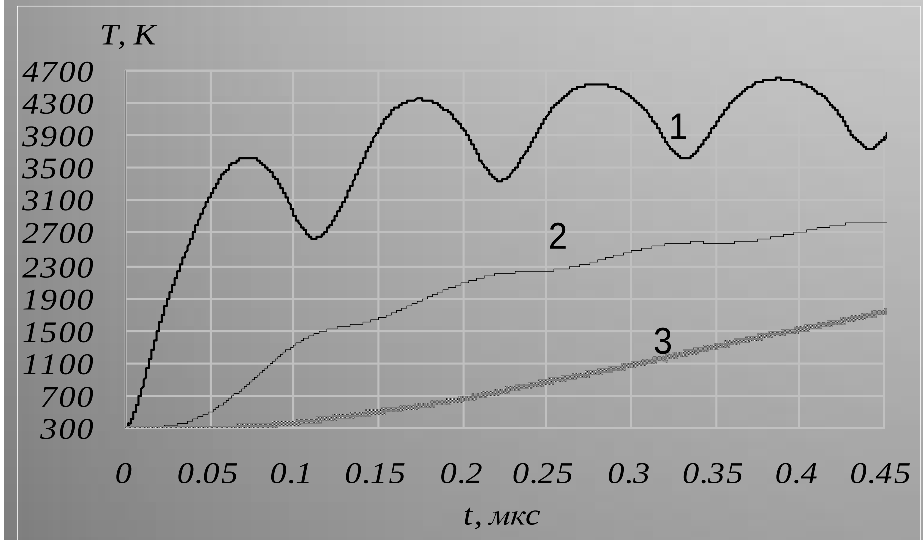
Solving the problem were obtained the following results for system of Fe – Ti



a

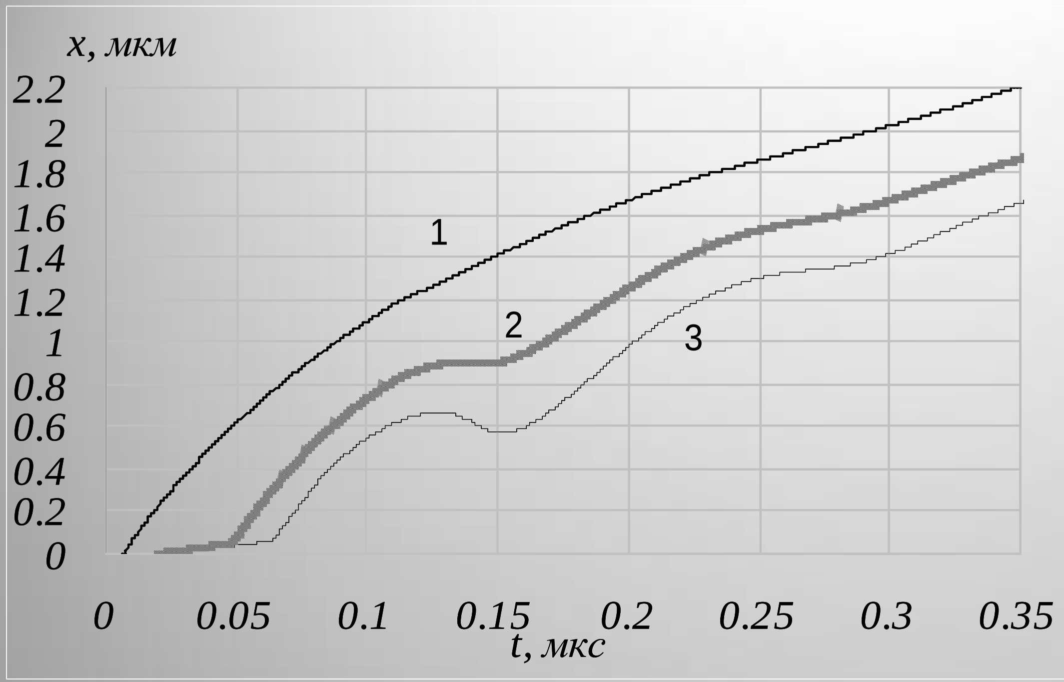


b



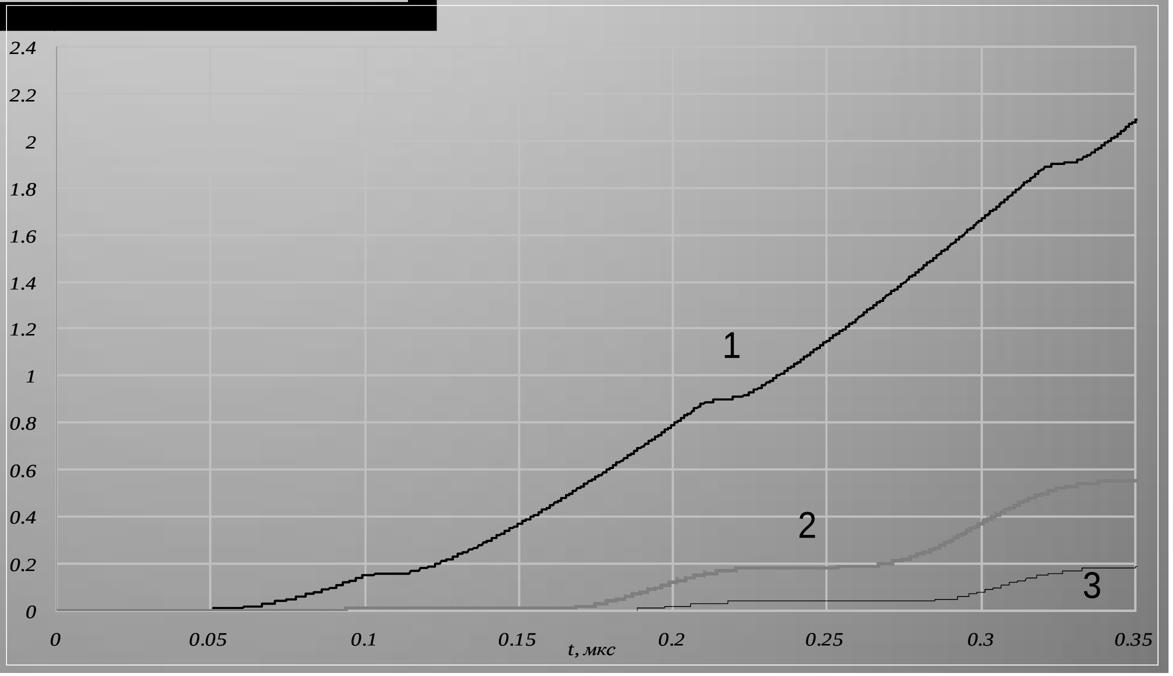
c

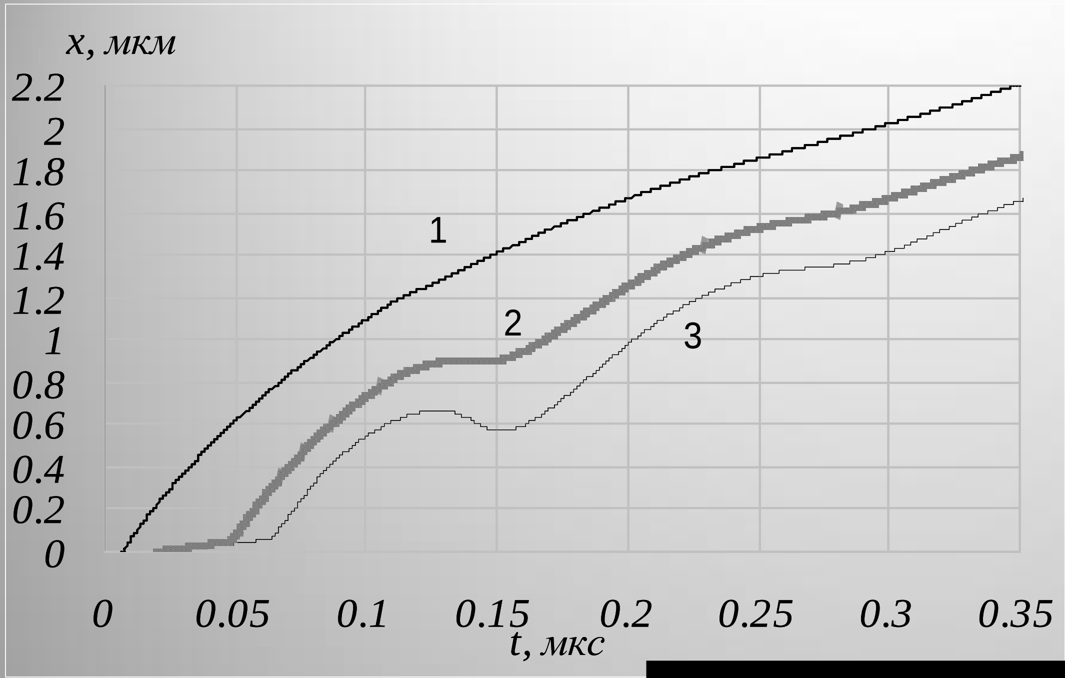
On the graphs there are presented the temperature fields in x=0 – curve 1, x=1.25 mcm – curve 2, x=2.5 – curve 3 mcm for a – pulse of “bench” type, b - pulse of a triangle form, c - sinusoidal pulse.



On the graph there is presented the front-line evaporation for 1 – pulse of “bench” type, 2 - pulse of a triangle form, 3 - sinusoidal pulse.

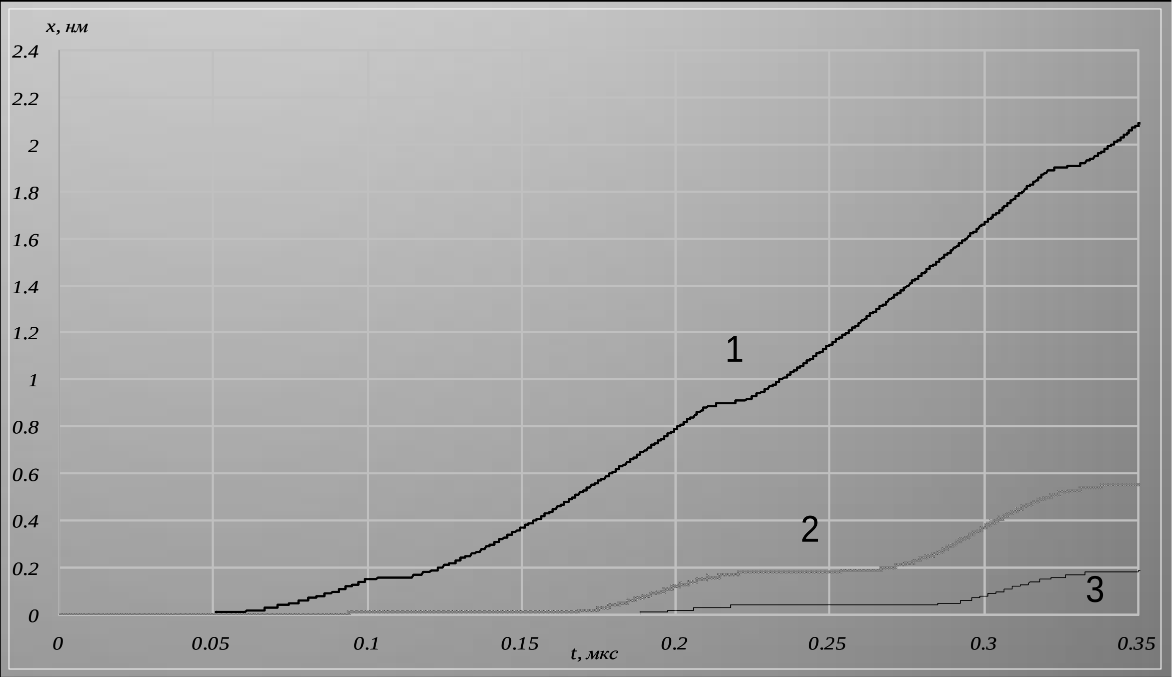
On the graph there is presented the Front-line motion for 1 – pulse of “bench” type, 2 - pulse of a triangle form, 3 - sinusoidal pulse.



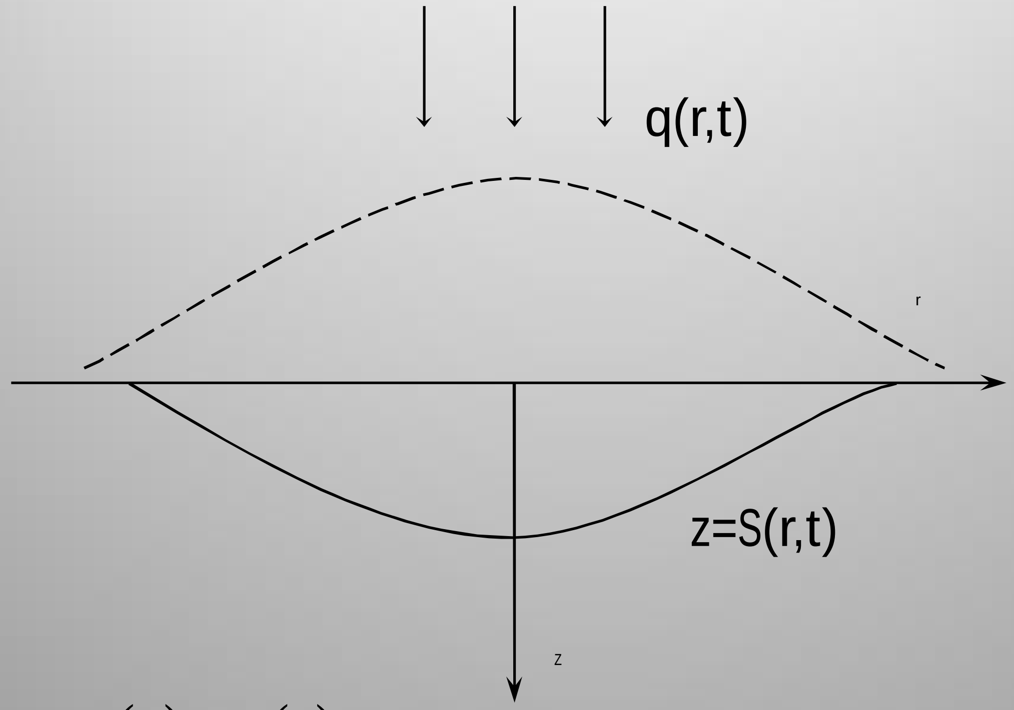


On the graph there is presented the Front-line motion for 1 – pulse of “bench” type, 2 - pulse of a triangle form, 3 - sinusoidal pulse.

On the graph there is presented the front-line evaporation for 1 – pulse of “bench” type, 2 - pulse of a triangle form, 3 - sinusoidal pulse.



The following phase of the research was solution of the two-dimensional direct problem



$$q(r,t) = q(r) \cdot q(t), \quad \text{where} \quad q(r) = q_{max} \cdot \exp(-k \cdot r^2),$$

k - concentration coefficient,

q(t) depends on the type of pulse.

Mathematic problem formulation

$$\rho(T)\left((c(T)+L\delta(T-T_n)\right)\frac{\partial T(z,r,t)}{\partial t}=-\left(\frac{1}{r}\frac{\partial}{\partial r}(r\cdot W_r)+\frac{\partial}{\partial z}(W_z)\right),$$

where $W_r = -\lambda(T)\frac{\partial T(z,r,t)}{\partial r}, 0<r<R$ $\quad W_z = -\lambda(T)\frac{\partial T(z,r,t)}{\partial z}, 0<z<Z$

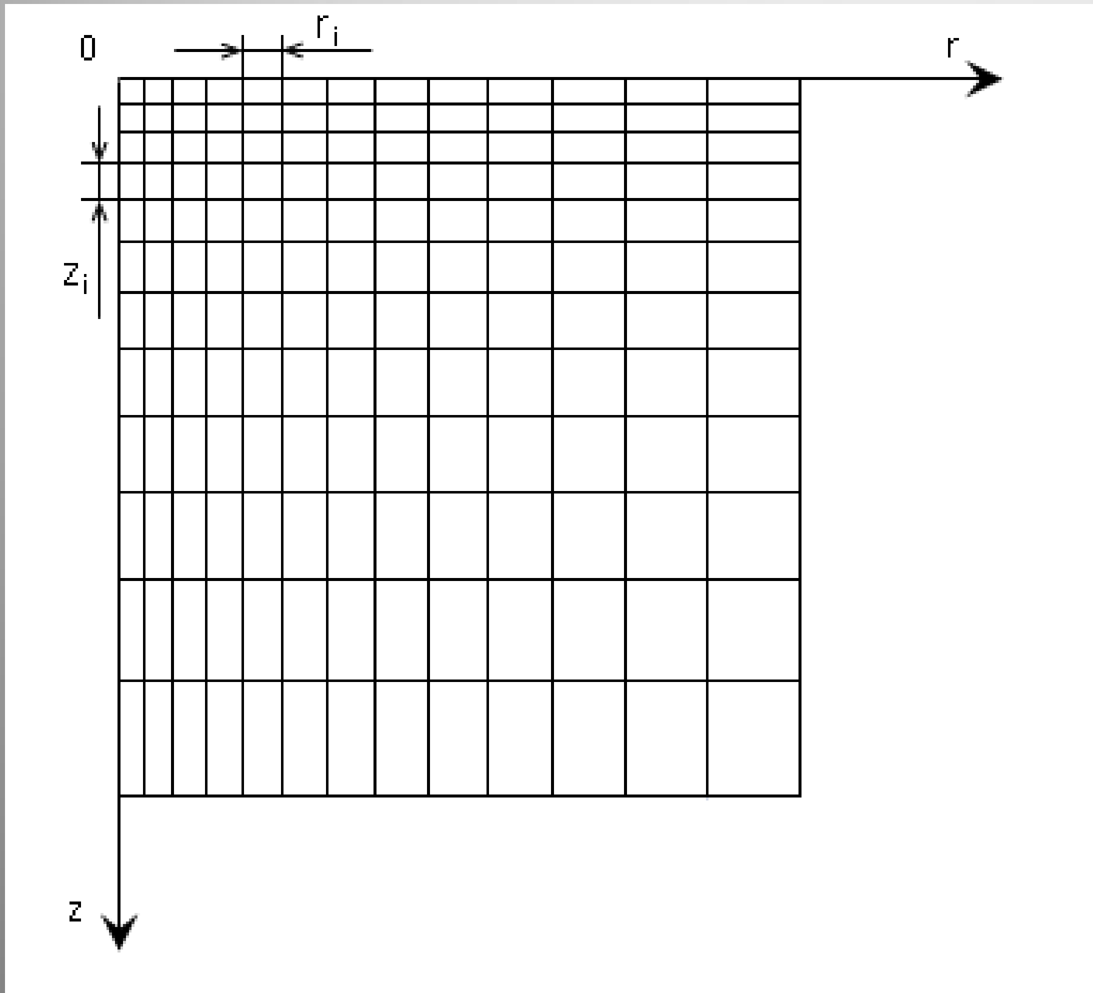
Initial and boundary conditions:

$$T(r,z,0)=T_0; \qquad -\lambda(T(z))\frac{\partial T(z)}{\partial z}\bigg|_{z=0}=q_1(t,r)$$

$$\lambda(T(z))\frac{\partial T(z)}{\partial z}\bigg|_{z=Z}=0 \qquad \lambda(T(r))\frac{\partial T(r)}{\partial r}\bigg|_{r=0}=0$$

$$\lambda(T(r))\frac{\partial T(r)}{\partial r}\bigg|_{r=R}=0$$

Method of solving the problem



$$r_i = i \cdot r_0; \quad i = \overline{1, N2}$$

$$z_j = j \cdot z_0; \quad j = \overline{1, N1}$$

where

$$r_0 = \frac{2 \cdot R}{N2 \cdot (N2 + 1)}$$

$$z_0 = \frac{2 \cdot Z}{N1 \cdot (N1 + 1)}$$

Method of solving the problem

$$\rho_1 \tilde{c}_1(T_1) = \rho_1(T)\left(c_1(T_1) + L_1 \delta(T_1 - T_{n,1^*}, \Delta)\right)$$

where $\rho_1 \tilde{c}_1(T_1) = c_{1,\text{solid}}(T_1)\rho_{1,\text{solid}}(T_1)$ when $T_1 < T_{n,1} - \Delta,$

$\rho_1 \tilde{c}_1(T_1) = c_{1,\text{liquid}}(T_1)\rho_{1,\text{liquid}}(T_1)$ when $T_1 > T_{n,1} + \Delta,$

and when ($T_{n,1}$ - Δ, $T_{n,1}$ + Δ)

$$\int_{T_{n,1}-\Delta}^{T_{n,1}+\Delta} \rho_1 \tilde{c}_1(T_1)dT_1 = \rho_1 L_1 + \int_{T_{n,1}-\Delta}^{T_{n,1}} \rho_{1,\text{т}}(T_1)c_{1,\text{т}}(T_1)dT_1 + \int_{T_{n,1}}^{T_{n,1}+\Delta} \rho_{1,\text{ж}}(T_1)c_{1,\text{ж}}(T_1)dT_1,$$

Method of solving the problem

$$\rho c_{i,j}^{k-1}\frac{U_{i,j}^{k}-T_{i,j}^{k-1}}{\tau}=\frac{2}{h_i+h_{i+1}}\left[\lambda_{i+\frac{1}{2},j}^{k-1}\frac{U_{i+1,j}^{k}-U_{i,j}^{k}}{h_{i+1}}-\lambda_{(i-1)+\frac{1}{2},j}^{k-1}\frac{U_{i,j}^{k}-U_{i-1,j}^{k}}{h_i}\right],$$

for $j=0,1,\ldots,N1,$

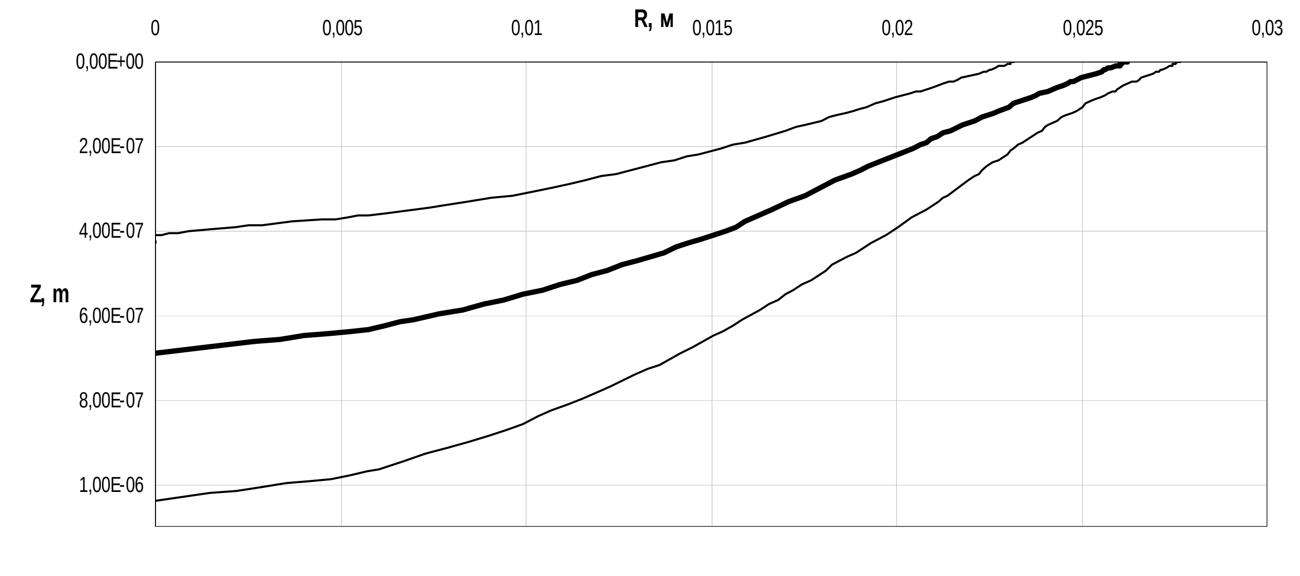
$i=1,\ldots,N2-1$

$$\rho c_{i,j}^{k-1}\frac{T_{i,j}^{k}-U_{i,j}^{k-1}}{\tau}=\frac{2}{\left(R_{j+1/2}^{2}-R_{j-1/2}^{2}\right)}\left[R_{j-1/2}\lambda_{j-1/2}^{k-1}\frac{T_{i,j-1}^{k}-T_{i,j}^{k}}{R_j-R_{j-1}}-R_{j+1/2}\lambda_{j+1/2}^{k-1}\frac{T_{i,j}^{k}-T_{i,j+1}^{k}}{R_{j+1}-R_j}\right],$$

for $i=0,1,\ldots,N2,$

$j=1,\ldots,N1-1$

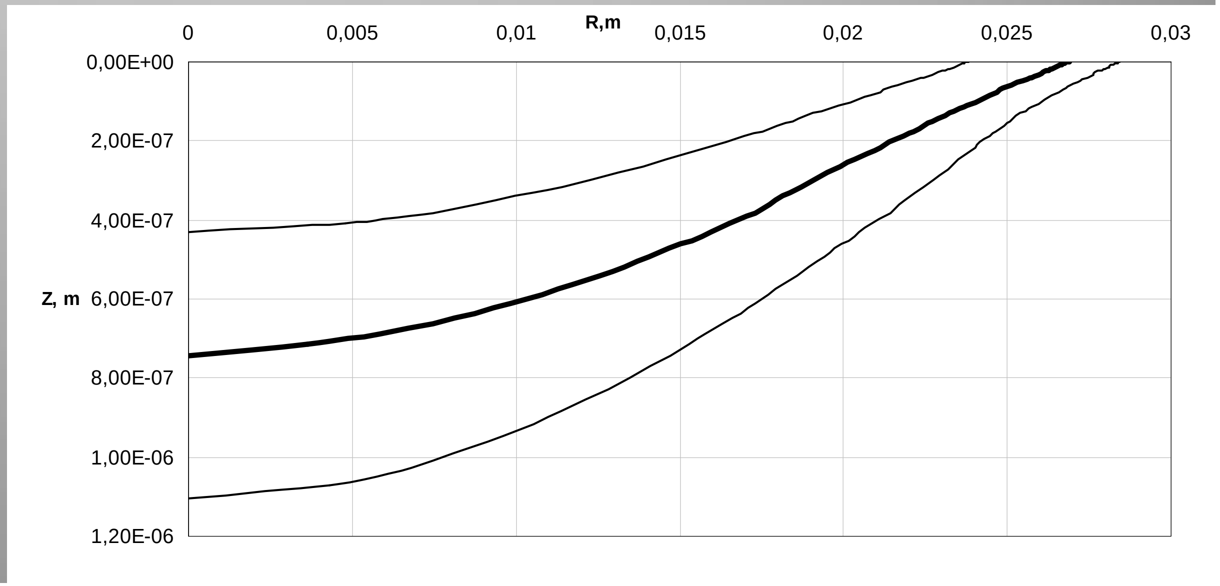
Solving the problem the following results were obtained

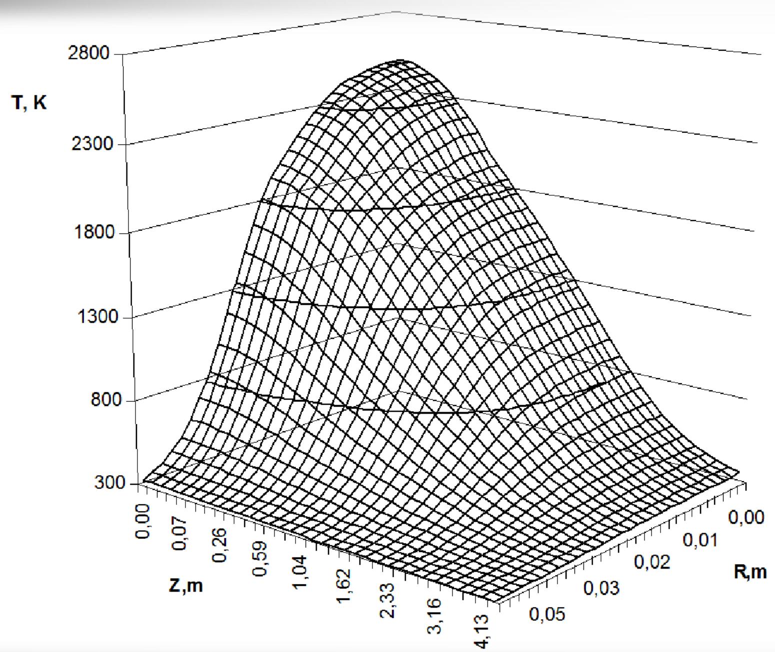


←

Front of material melting for pulse of a triangle form *at t= 0.17mcs – curve 1; 0.27msc – curve 2; t = 0.37msc – curve 3* .1 – pulse of “bench” type, 2 -, 3 - sinusoidal pulse.

Front of material melting for sinusoidal pulse *at t= 0.17mcs – curve 1; 0.27msc – curve 2; t = 0.37msc – curve 3 .*





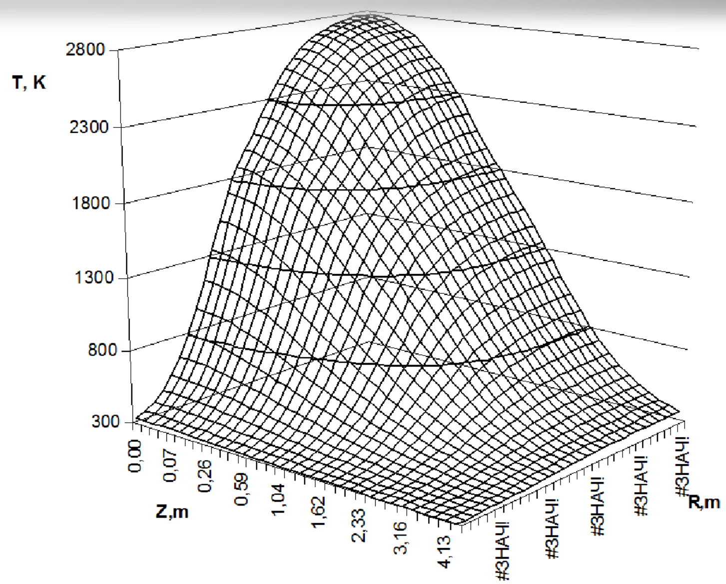
a

↑

Temperature distribution in the system when t= 0.27msc a – triangle pulse,

Temperature distribution in the system when t= 0.27msc – b- sinusoidal pulse

↓

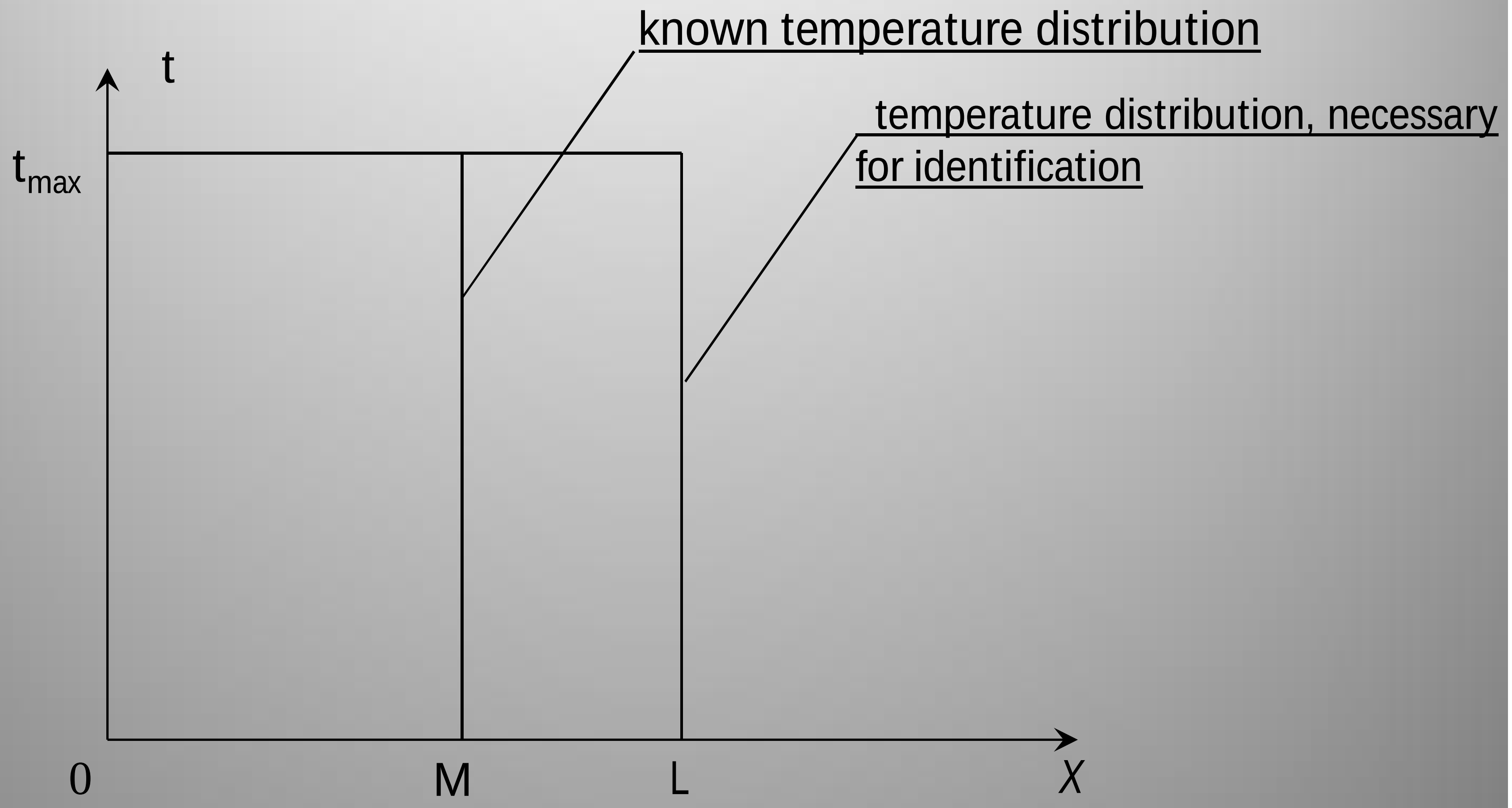


b

The following phase of research work was the transition from solution of the direct problems to solution of the inverse problems

Thanks to this action there appears possibility to create mathematical model of strengthening surface with laser flow in order to determine optimal conditions of heating, hence we will look for the solution of the problem of control.

This problems can be represent the following way:



Mathematic statement of the problem

$$c\rho\frac{\partial T}{\partial t}=\lambda\frac{\partial^2 T}{\partial x^2},\quad x\in(0,M)\cup(M,L)<L,\ 0<t<\tau$$

Initial and boundary conditions:

$$\lambda\frac{\partial T}{\partial x}\bigg|_{x=0}=0\qquad T(M,t)=\varphi(t)\qquad T(x,0)=T_0\qquad 0<M<L$$

Transition this problem to dimensionless form

$$\frac{\partial\Theta}{\partial Fo}=\frac{\partial^2\Theta}{\partial X^2},\quad X\in\left(0,{}^{M}\!/\!{}_{L}\right)\cup\left({}^{M}\!/\!{}_{L},1\right),\ 0<Fo<Fo_{\max}$$

Initial and boundary conditions:

$$\frac{\partial\Theta}{\partial X}\bigg|_{X=0}=0\quad \Theta\left({}^{M}\!/\!{}_{L},Fo\right)=\varphi^*(Fo)\qquad \Theta(X,0)=1\qquad 0<{}^{M}\!/\!{}_{L}<1$$

The first stage: this is solution of the direct problem in the following way

$$\frac{\partial \Theta}{\partial Fo} = \frac{\partial^2 \Theta}{\partial X^2}, \qquad X \in \left(0, {M}/{L}\right),\ 0 < Fo < Fo_{\max}$$

$$\left.\frac{\partial \Theta}{\partial X}\right|_{X=0} = 0 \qquad \Theta\left({M}/{L}, Fo\right) = \varphi^*(Fo) \qquad \Theta(X,0) = 1 \quad 0 < X < {M}/{L}$$

The second stage:

$$\frac{\partial \Theta}{\partial Fo} = \frac{\partial^2 \Theta}{\partial X^2}, \qquad X \in \left({M}/{L}, 1\right),\ 0 < Fo < Fo_{\max}$$

$$\left.\frac{\partial \Theta}{\partial X}\right|_{X={M}/{L}} = Ki(Fo) \quad \Theta\left({M}/{L}, Fo\right) = \varphi^*(Fo) \quad \Theta(X,0) = 1 \quad 0 < {M}/{L}$$

For solving the problem with quasiinvertion's method the following equation is obtained:

$$\frac{d^2\Theta_\alpha}{dX^2} - AU_\alpha + \alpha AA^* \frac{d\Theta_\alpha}{dX} = 0$$

where $A = \partial/\partial Fo$; $A = \partial/\partial Fo$; α – regularization parameter

hence

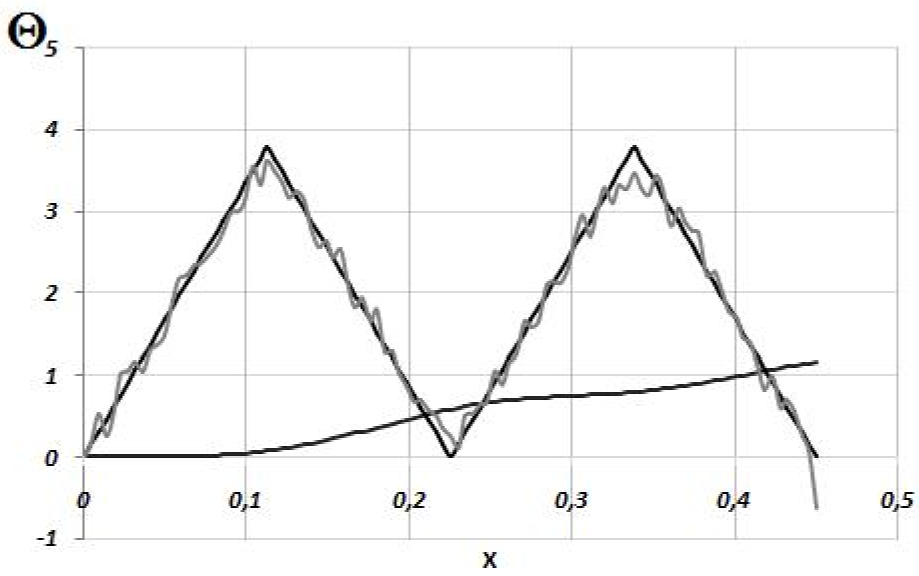
$$\frac{\partial^2\Theta_\alpha}{\partial X^2} - \frac{\partial\Theta_\alpha}{\partial Fo} - \alpha \frac{\partial^3\Theta_\alpha}{\partial Fo^2 \partial X} = 0, \ \ M/L < X < 1, \ 0 < Fo < Fo_{\max}$$

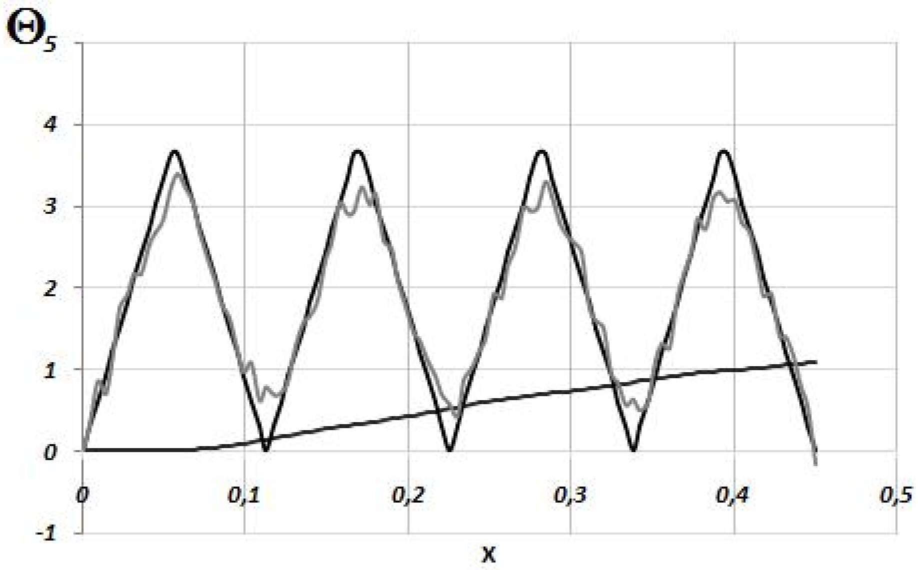
This equation is completed by the following boundaries and initial conditions

$$\Theta_\alpha\left(M/L, Fo\right) = \varphi_M(Fo), \ 0 < Fo < Fo_{\max} \quad \frac{\partial\Theta_\alpha}{\partial X}\left(M/L, Fo\right) = -Ki(Fo), \ 0 < Fo < Fo_{\max}$$

$$\Theta_\alpha(x,0) = 1, \ 0 < X < 1 \quad \frac{\partial\Theta_\alpha}{\partial Fo}(X,0) = 0, \ 0 < X < 1$$

Solving the problem were obtained the following results





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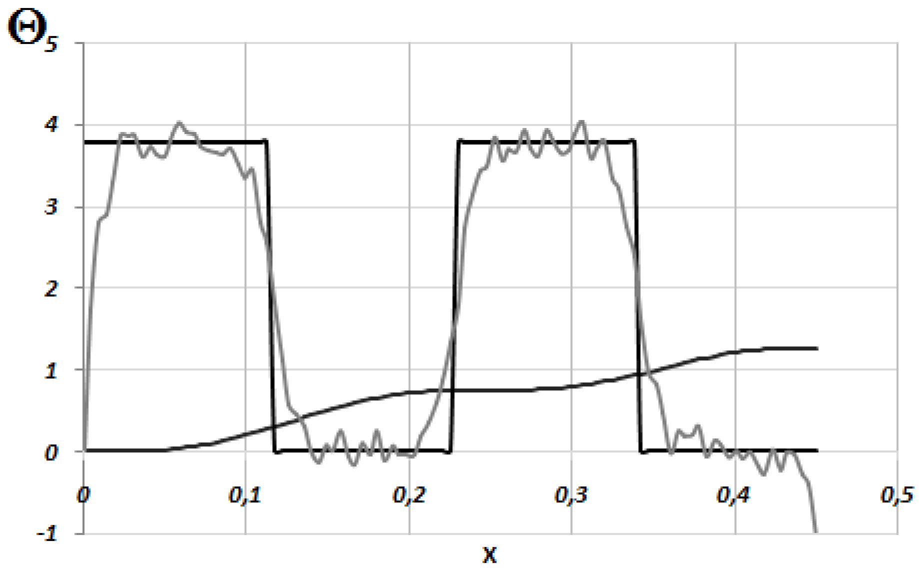

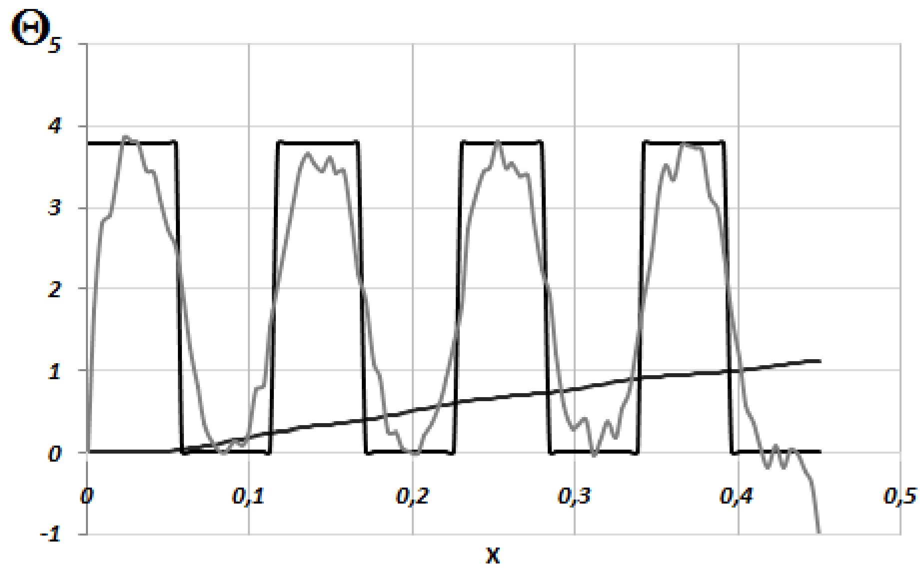

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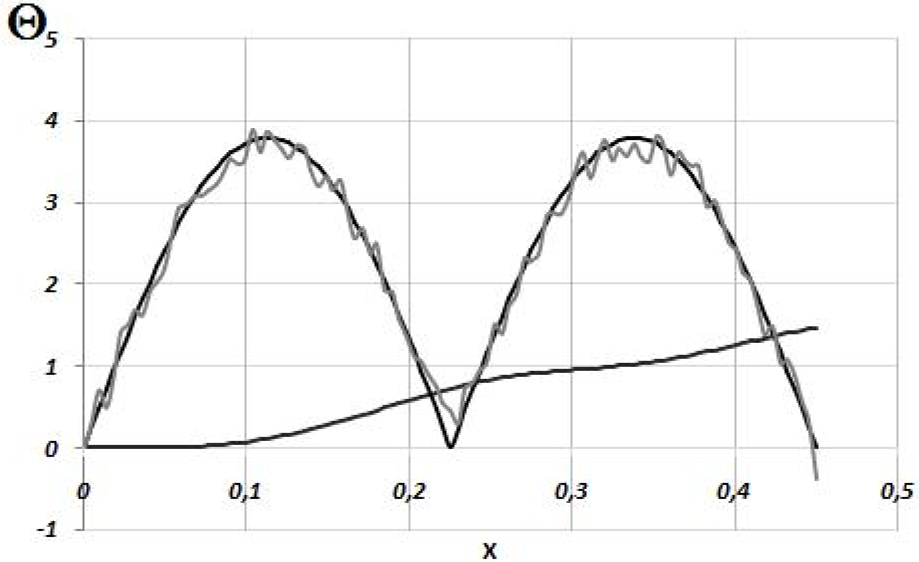

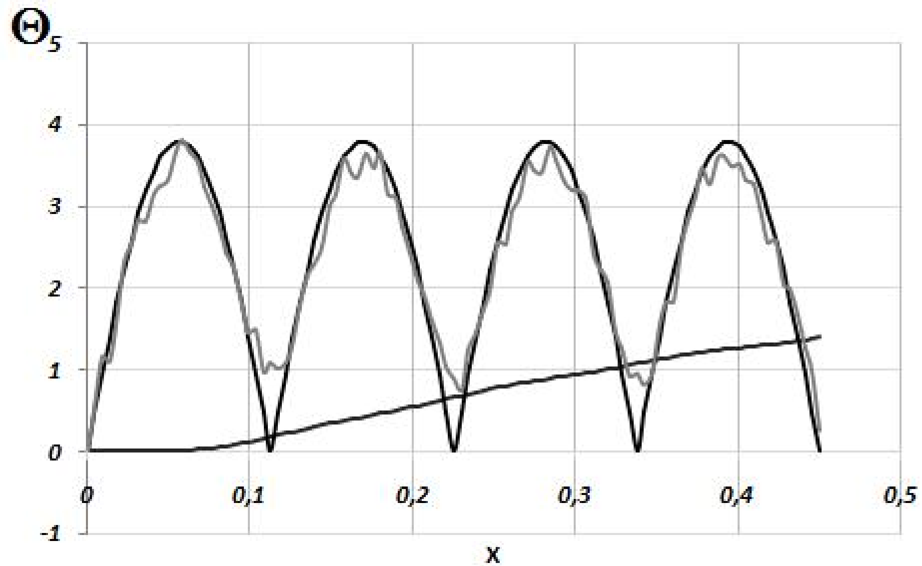

Thank you for your attention!