Trigonometria MATEMATIIKAN KAAVOJA 11/2006

(1) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta,\qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

(2) $\tan(\alpha \pm \beta) = (\tan\alpha \pm \tan\beta)/(1 \mp \tan\alpha\tan\beta)$

(3) $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2},\qquad \sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

(4) $\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2},\qquad \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

(5) $\sin\alpha\sin\beta = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta),\qquad \cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$

(6) $\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta)$

(7) $1 + \tan^2\alpha = \frac{1}{\cos^2\alpha},\qquad 1 + \cot^2\alpha = \frac{1}{\sin^2\alpha},\qquad 1 + \cos\alpha = 2\cos^2\frac{\alpha}{2},\qquad 1 - \cos\alpha = 2\sin^2\frac{\alpha}{2}$

(8) $\sin 2\alpha = 2\sin\alpha\cos\alpha,\qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha,\qquad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

(9) $\sin\alpha = \frac{2t}{1+t^2},\ \cos\alpha = \frac{1-t^2}{1+t^2},\ \tan\alpha = \frac{2t}{1-t^2},$ missä $t = \tan\frac{\alpha}{2},\qquad \tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$

(10) $\sinh x = \frac{1}{2}(e^x - e^{-x}),\ \cosh x = \frac{1}{2}(e^x + e^{-x}),\ \operatorname{arsinh} x = \ln(x + \sqrt{x^2+1}),\ \operatorname{arcosh} x = \ln(x + \sqrt{x^2-1})$

(11) $\cosh^2 x - \sinh^2 x = 1,\qquad \sinh 2x = 2\sinh x\cosh x,\qquad \cosh 2x = \cosh^2 x + \sinh^2 x$

Sarjat

(12) $f(x) = \sum_{k=0}^{n-1}\frac{f^{(k)}(x_0)}{k!}(x-x_0)^k + \frac{f^{(n)}(\xi)}{n!}(x-x_0)^n$

(13) $(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \cdots,\qquad |x| < 1$

(14) $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{2\cdot 4}x^2 + \frac{1\cdot 3}{2\cdot 4\cdot 6}x^3 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}x^4 + \cdots,\qquad |x| \le 1$

(15) $\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1\cdot 3}{2\cdot 4}x^2 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3 + \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}x^4 - \cdots,\qquad -1 < x \le 1$

(16) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

(17) $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots,\qquad -1 < x \le 1$

(18) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots,\qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

(19) $\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots,\qquad |x| < \frac{\pi}{2}$

(20) $\arcsin x = x + \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{x^7}{7} + \cdots,\qquad |x| \le 1$

(21) $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots,\qquad |x| \le 1$

(22) $\frac{1}{2}\ln\frac{1+x}{1-x} = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots,\qquad |x| < 1$

(23) $(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k},\qquad \sum_{i=1}^{n} i = \frac{1}{2}n(n+1),\qquad \sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$

(24) $\frac{1}{(1-x)^n} = \sum_{k=0}^{\infty}\binom{n+k-1}{k}x^k,\quad |x| < 1, n \ge 1.$

Integrointi

(25) $\int Q(x,\sqrt{x^2+a^2})dx:$ sij. $x=at,\ a\tan t,\ a\sinh t,\ldots$ $\quad\int Q(x,\sqrt[n]{\frac{ax+b}{cx+d}})dx:$ sij. $t=\sqrt[n]{\frac{ax+b}{cx+d}}$

(26) $\int Q(x,\sqrt{x^2-a^2})dx:$ sij. $x=at,\ a/t,\ a\cosh t,\ldots$ $\quad\int Q(x,\sqrt{a^2-x^2})dx:$ sij. $x=at,\ a\sin t,\ldots$

(27) $\int Q(\tan x)dx:$ sij. $t=\tan x,\ t=\cot x,\ldots$ $\quad\int Q(\sin x,\cos x)dx:$ sij. $t=\tan\frac{x}{2},\ldots$

(28) $\displaystyle\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|,\qquad\int\frac{dx}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}$

(29) $\displaystyle\int\frac{dx}{(a^2\pm x^2)^{n+1}}=\frac{x}{2na^2(a^2\pm x^2)^n}+\frac{2n-1}{2na^2}\int\frac{dx}{(a^2\pm x^2)^n}$

(30) $\displaystyle\int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{|a|},\qquad\int\frac{dx}{\sqrt{x^2+a}}=\ln\left|x+\sqrt{x^2+a}\right|$

(31) $\displaystyle\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\arcsin\frac{x}{|a|}$

(32) $\displaystyle\int\sqrt{x^2+a}\,dx=\frac{x}{2}\sqrt{x^2+a}+\frac{a}{2}\ln\left|x+\sqrt{x^2+a}\right|$

(33) $\displaystyle\int\cos^2x\,dx=\frac{1}{2}x+\frac{1}{4}\sin 2x,\qquad\int\sin^2x\,dx=\frac{1}{2}x-\frac{1}{4}\sin 2x$

(34) $\displaystyle\int\frac{dx}{\cos x}=\ln\left|\tan(\frac{x}{2}+\frac{\pi}{4})\right|,\qquad\int\frac{dx}{\sin x}=\ln\left|\tan\frac{x}{2}\right|$

(35) $\displaystyle\int\frac{dx}{\cosh x}=2\arctan e^x,\qquad\int\frac{dx}{\sinh x}=\ln\left|\tanh\frac{x}{2}\right|$

(36) $\displaystyle\int e^{ax}\sin bx\,dx=\frac{a\sin bx-b\cos bx}{a^2+b^2}e^{ax},\qquad\int e^{ax}\cos bx\,dx=\frac{a\cos bx+b\sin bx}{a^2+b^2}e^{ax}$

(37) $\displaystyle\int_0^{\pi/2}\sin^{2k}x\,dx=\int_0^{\pi/2}\cos^{2k}x\,dx=\frac{1}{2}\cdot\frac{3}{4}\cdots\frac{2k-1}{2k}\cdot\frac{\pi}{2}$

(38) $\displaystyle\int_0^{\pi/2}\sin^{2k+1}x\,dx=\int_0^{\pi/2}\cos^{2k+1}x\,dx=\frac{2}{3}\cdot\frac{4}{5}\cdots\frac{2k}{2k+1}$

(39) Kaaren pituus $\displaystyle\int\sqrt{1+y'^2}dx=\int\sqrt{x'^2+y'^2}dt=\int\sqrt{r^2+r'^2}d\varphi$

(40) Pyöräyspinnan ala $\displaystyle 2\pi\int|y|\sqrt{1+y'^2}dx$

(41) $\displaystyle\int xe^{-ax}dx=-\frac{e^{-ax}}{a}\left(x+\frac{1}{a}\right),\qquad\int x^2e^{-ax}dx=-\frac{e^{-ax}}{a}\left(\left(x+\frac{1}{a}\right)^2+\frac{1}{a^2}\right)$

(42) $\displaystyle\int_0^\infty x^ne^{-ax}dx=\frac{n!}{a^{n+1}},\quad n=0,1,2,\ldots,\qquad\int_{-\infty}^\infty e^{-ax^2}dx=\sqrt{\frac{\pi}{a}}$

(43) $\displaystyle\int_{-\infty}^\infty x^{2n}e^{-ax^2}dx=\frac{(n-\frac{1}{2})(n-\frac{3}{2})\cdots\frac{1}{2}}{a^n}\sqrt{\frac{\pi}{a}},\quad n=1,2,\ldots$

(44) $\displaystyle\frac{\partial}{\partial z}\int_{\alpha(z)}^{\beta(z)}f(x,z)dx=\int_{\alpha(z)}^{\beta(z)}\frac{\partial f}{\partial z}dx+f(\beta(z),z)\frac{\partial\beta}{\partial z}-f(\alpha(z),z)\frac{\partial\alpha}{\partial z}$