Abstract

After a brief description of a representative selection of power indices and a discussion of the notion of power in collective decision making, the paper discusses the modelling of power of an individual or collective agent as identified with the potential or factual effect the decision of this agent has on the outcome. It demonstrates that the distribution of power is crucially dependent on the procedures resorted to, and not just on the distribution of resources and the majority threshold as captured by the standard power measures. Similarly, it is shown that the selection and formulation of the questions to analyze can be highly relevant when we link power and power measures to causality. The concluding section discusses whether power indices are measures that represent power and ratios of power, or whether they are indicators that point out properties of the cooperative game and the underlying decision situation.

Keywords: causality, power indices, preference proximity

1 Introduction

Open problems in the analysis of power sharing in politics can in general be traced back to the problem that we do in fact not know what power is.
Power is a theoretical construction and is defined by the questions that we analyze. We have no straightforward access to it that derives from empirical facts. We need theorizing and reduction.

When it comes to politics and within politics to voting there is a widely shared agreement that the power of an agent \( i \) (an individual voter, party or a faction) can be expressed by the potential that \( i \) can change the outcome. This potential then is equated with the probability that \( i \) can change a winning coalition into a losing one, and vice versa. This suggests to look for a probability interpretation of concepts that serve as solution to the power sharing problem, i.e., to the cooperative games that are used to models it. However, is power a probability or chance? The latter concurs with Max Weber’s definition of power although its English translation uses “probability” for the German “Chance” [see Weber, 1947: 152]). Holler and Nurmi [2010] conclude that it could be helpful to express power in terms of probability [see Brückner, 2002, and Widgrén, 2002], but to consider the two concepts identical misses the point.

Felsenthal and Machover [1998] distinguish I-power and P-power when decisions are made by voting. I-power is based on policy-seeking and hence proportional to the voter’s influence on the voting outcome, while P-Power refers to sharing the spoils that go to the winning coalition, e.g., cabinet positions. As a consequence, they allocate the various power measures that are introduced below to these categories. This could help to answer the question of which power measure to choose. However, Turnovec [2004] argues that the formal structure of the various indices does not justify this categorization.

Of course, the multitude of power measures, pointed out in section 2, is another issue that is considered an open problem: Which index is the right one? A possible answer is due to Robert Aumann [1977: 464):

None of them; they are all indicators, not predictors. Different solution concepts are like different indicators of an economy; different methods for calculating a price index; different maps (road, topo, political, geologic, etc., not to speak of scale, projection, etc.); different stock indices (Dow Jones They depict or illuminate the situation from different angles; each one stresses certain aspects at the expense of others.

We subscribe to this perspective. In section 6, we will discuss whether power indices are measures that represent power and ratios of power, or whether they are indicators that point out properties of the cooperative game and the underlying decision situation. First, however, in section 2 we will briefly describe selected power indices. In general, the power of an individual or collective agent is by and large identified with the potential or factual effect the decision of this agent has on the outcome. Section 3 discusses the modelling of these effects. Section 4 argues that the distribution of power
is crucially dependent on the procedures resorted to, not just on the distribution of resources and majority threshold. Section 5 illustrates some problems that result when we link power and power measures to causality.

2 Power Indices

Several power indices have been suggested in the literature. While not equivalent, some of them share important properties [see Allingham 1975; Freixas and Gambarelli 2008]. However, there remains the question what index to choose.

The Shapley-Shubik index is a projection of the Shapley value to simple games [Shapley 1953]. The corresponding value of player \( i \) in a game of \( n \) players is:

\[
\phi_i = \sum_{S \subseteq N} \frac{(s-1)! (n-s)!}{n!} [v(S) - v(S \setminus \{i\})].
\]

Here \( N \) denotes the set of players, \( s \) the number of members of coalition \( S \) and \( n! \) is defined as the product \( n \times (n-1) \times (n-2) \times 2 \times 1 \). The expression in square brackets differs from zero just in case \( S \) is winning but \( S \setminus i \) is not. In this case, then, \( i \) is a decisive member in \( S \). In other words, \( i \) has a swing in \( S \).

Indeed, the Shapley-Shubik index value of \( i \) indicates the expected share of \( i \)'s swings in all swings assuming that coalitions are formed sequentially. The Shapley-Shubik index can be viewed as a measure based on the assumption that all attitude dimensions (e.g. sequences of actors in order from the most supportive to the least supportive one) are equiprobable.

The two indices named after Banzhaf replace this equiprobability of dimensions assumption with one that pertains to coalitions [Banzhaf 1965]. The standardized Banzhaf index value of \( i \) is defined as:

\[
\bar{\beta}_i = \frac{\sum_{S \subseteq N} [v(S) - v(S \setminus \{i\})]}{\sum_{j \in N} \sum_{S \subseteq N} [v(S) - v(S \setminus \{j\})]}.
\]

The absolute Penrose-Banzhaf index [Penrose 1946; Banzhaf 1965], in turn, is defined as:

\[
\beta_i = \frac{\sum_{S \subseteq N} [v(S) - v(S \setminus \{i\})]}{2^{n-1}}.
\]

In other words, this index counts the number of swings and divides this by the number of coalitions where the actor is present. In contrast to the previous ones, the values of absolute Penrose-Banzhaf index, when summed over the actors, do not in general add up to unity.

Two more recent measures, viz. the Public Good Index [Holler 1982] and Deegan-Packel [1979] index, focus on minimal winning coalitions, i.e.
on coalitions in which all actors are decisive in the sense that should any one of them leave the coalition, it would become a losing coalition. The importance of players, and consequently their payoff expectation, is reflected by the number of presences in these types of coalitions.

Player $i$’s Public Goods Index value $h_i$ is computed as follows:

$$h_i = \frac{\sum_{S \subseteq N}[v(S) - v(S \setminus \{i\})]}{\sum_{j \in N} \sum_{S \subseteq N}[v(S) - v(S \setminus \{j\})]}.$$  

Here $S*$ is a minimal winning coalition. The Deegan-Packel index value of player $i$, denoted $DP_i$, in turn, is obtained as follows:

$$DP_i = \frac{\sum_{S \subseteq N} 1/s[v(S) - v(S \setminus \{i\})]}{\sum_{j \in N} \sum_{S \subseteq N} 1/s[v(S) - v(S \setminus \{j\})]}.$$ 

This measure contains the explicit sharing rule $1/s$.

Bertini at al. [2008] suggest a Public Help Index (PHI) that shares the public good idea with the Public Good Index (PGI), however, it takes into consideration that a dummy may also profit from the provision of a public good. As a consequence the PHI is not restricted to minimal winning coalitions. A possible interpretation is that the PHI considers the consumption of public goods, taking care of non-exclusion, while the PGI focuses on their provision. In the latter case, surplus players invite free-riding and the provision of the public good becomes unlikely or a matter of “luck”. Holler (1982) proposes that we should consider minimum winning coalitions only when it comes to measuring power and the outcome is a public good. This is not to say that merely minimum winning coalitions will form.

The above measures have been axiomatized using various sets of axioms. As Aumann [1977: 471] observes:

“axiomatization underscores the fact that a ‘perfect’ solution concept is an unattainable goal, a fata morgana; there is something ‘wrong’, some quirk with every one.”

Still, axiomatizations serve a number of useful purposes. First, like any other alternative characterization, they shed additional light on a concept and enable us to ‘understand’ it better. Second, they underscore and clarify important similarities between concepts, as well as differences between them.

It is noteworthy that monotonicity, however defined, is not explicit in the axioms. However, when discussing the properties of the measures then the various concepts of monotonicity seem to be essential. We come back to this discussion in section 6.
3 The Influence over Outcomes

A problem often faced in institution design is how to share the voting power so that it would coincide with a given distribution of power resources (say, seats in the parliament). With more than three alternatives, it turns out that there are settings where a player would have more control over the outcome with less resources than with more of them [see Holler and Nurmi 2011]. In what follows we show that this possibility also emerges in spatial voting games, i.e. it may happen that a small group of players has more influence over outcomes than a bigger group. Hence, were we to share power in proportion to resources, we would occasionally need to give small groups larger shares than bigger groups. This perhaps counterintuitive observation is based on a theorem proven by Baigent [1987] and strengthened by Eckert [2004]. We shall illustrate it with an example.

Consider a voting body and a very small group of voters with identical preferences in it. Suppose that the voters make a mistake in reporting their preferences in an election. One of the group members may have interpreted the content of decision alternatives incorrectly and the others are following his lead in reporting their preferences in voting. Since we are dealing with a small group of voters, the preference profile containing the intended preferences and the one containing the erroneous preferences should be – if not identical – close to each other. Now, a plausible desideratum for a voting procedure is that mistakes of small voter groups and the accompanying small changes in preference profiles should not result in large changes in ensuing voting outcomes. In particular, the changes in the latter should not be larger as a result of mistaken reports of small voter groups than as a result of mistakes of larger ones. This is intuitively what voting power is about: changing the ballots of big groups should make a larger difference in voting outcomes than changing the ballots of small groups. This prima facie plausible desideratum turns, however, out to be incompatible with other intuitively compelling requirements of social choices.

The fundamental results in this area are due to Baigent [1987]. For an illustration we modify an example given in Baigent’s paper. Consider a drastic simplification of policy options of the member countries of the Euro zone in the early fall of 2011 regarding the imminent debt crisis of Greece. Let us assume that there are only two Euro zone countries (country 1 and country 2) and two policy alternatives: to establish a massive bailout programme based on Euro bonds to rescue the Greek economy (bailout) or to refrain from bailout activities (default). We assume that both countries have strict preference with regard to these options. Four profiles are now possible as listed in the following table:

We denote the voters’ rankings in various profiles by $P_{mi}$ where $m$ denotes the profile and $i$ denotes the voter. $k$ and $j$ are particular values of $m$ such that $k \neq j$. $N$ is the set of voters. We consider two types of metrics:
Table 1: Stylized Greek Bailout Example

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th></th>
<th>$P_2$</th>
<th></th>
<th>$P_3$</th>
<th></th>
<th>$P_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>bailout</td>
<td>2</td>
<td>default</td>
<td>2</td>
<td>default</td>
<td>1</td>
<td>bailout</td>
</tr>
<tr>
<td>2</td>
<td>default</td>
<td>2</td>
<td>bailout</td>
<td>bailout</td>
<td>default</td>
<td>bailout</td>
<td>default</td>
</tr>
</tbody>
</table>

$d_r$ is defined on pairs of rankings and $d_P$ refers to profiles. The former is denoted by $d_r$ and the latter by $d_P$. The two metrics are related as follows:

$$d_P(P_k, P_j) = \sum_{i \in N} d_r(P_{ki}, P_{ji}).$$

In other words, the distance between two profiles $P_k$ and $P_j$ is the sum of distances between the pairs of rankings of the first, second, etc. voters. No further assumptions on the metric have been made. Take now two profiles, $P_1$ and $P_3$, from the above table and express their distance using metric $d_P$ as follows:

$$d_P(P_1, P_3) = d_r(P_{11}, P_{31}) + d_r(P_{12}, P_{32}).$$

Since, $P_{12} = P_{32} = \text{bailout} > \text{default}$, and hence the latter summand equals zero, $d_P(P_1, P_3)$ reduces to: $d_P(P_1, P_3) = d_r(P_{11}, P_{31}) = d_r(\langle \text{bailout} > \text{default} \rangle, \langle \text{default} > \text{bailout} \rangle)$. 

Taking now the distance between $P_3$ and $P_4$, we get:

$$d_P(P_3, P_4) = d_r(P_{31}, P_{41}) + d_r(P_{32}, P_{42}).$$

Both summands are equal since by definition: $d_r(\langle \text{default} > \text{bailout} \rangle, \langle \text{bailout} > \text{default} \rangle) = d_r(\langle \text{bailout} > \text{default} \rangle, \langle \text{default} > \text{bailout} \rangle)$. Thus, $d_P(P_3, P_4) = 2 \times d_r(\langle \text{bailout} > \text{default} \rangle, \langle \text{default} > \text{bailout} \rangle)$, i.e., in terms of $d_P$, then, $P_3$ is closer to $P_1$ than to $P_4$. This makes sense intuitively.

The proximity of the social choices emerging out of various profiles depends on the applied choice procedure $g$. Let us make two very mild restrictions on choice procedures, viz. that they are anonymous and respect unanimity. The former states that the choices are not dependent on the labelling of the voters. The latter, in turn, means that if all voters agree on a preference ranking, then that ranking is chosen. In our example, anonymity requires that whatever is the choice in $P_3$ is also the choice in $P_4$ since these two profiles can be reduced to each other by relabelling the voters. Unanimity, in turn, requires that $g(P_1) = \text{bailout}$, while $g(P_2) = \text{default}$. Therefore, either $g(P_3) \neq g(P_1)$ or $g(P_3) \neq g(P_2)$. Let’s assume the former. It then follows that $d_r(g(P_3), g(P_1)) > 0$. Recalling the implication of anonymity, we now have:
\[ d_r(g(P_3), g(P_1)) > 0 = d_r(g(P_3, g(P_4))). \]

In other words, even though \( P_3 \) is closer to \( P_1 \) than to \( P_4 \), the choice made in \( P_3 \) is closer to – indeed identical with – that made in \( P_4 \). This argument rests on the assumption that \( g(P_3) \neq g(P_1) \). Similar argument can be made for the alternative assumption, viz. that \( g(P_3) \neq g(P_2) \). The example thus shows that anonymity and respect for unanimity cannot be reconciled with a property called proximity preservation [Baigent 1987; Baigent and Klamler 2004]: choices made in profiles more close to each other ought to be closer to each other than those made in profiles less close to each other.

The example demonstrates that small mistakes or errors made by voters are not necessarily accompanied by small changes in voting outcomes. Indeed, if the true preferences of voters are those of \( P_3 \), then voter 1’s mistaken report of his preferences leads to profile \( P_1 \), while both voters’ making a mistake leads to \( P_4 \). Yet, the outcome ensuing from \( P_1 \) is further away from the outcome resulting from \( P_3 \) than the outcome that would have resulted had more, i.e., both, voters made a mistake (whereupon \( P_4 \) would have emerged). The above example illustrates that the mistakes of voters can make a substantial difference. It should be emphasized that the violation of proximity preservation occurs in a wide variety of voting systems, viz. those that satisfy anonymity and unanimity. This result is not dependent on any particular metric with respect to which the distances between profiles and outcomes are measured. Therefore we can conclude that in nearly all reasonable voting systems it is possible that a smaller group of voters has a greater impact on voting outcomes than a larger group. Thus, we have case of a violation of local monotonicity (LM).

4 Procedure and Control over Outcomes

The classic power indices as well as their more recently defined spatial competitors focus on dichotomous (yes-no) voting situations. This is, indeed, appropriate in many voting contexts. But it is equally reasonable to realize that there is an abundance of non-dichotomous choice situations. Once these are encountered, new problems of a priori voting power measurement arise [see Nurmi 2010]. Consider a voting body of nine individuals electing the president of the body. There are three candidates: \( A, B \) and \( C \). The members represent three different parties \( I, II \) and \( III \) so that 4 members belong to party \( I \), 3 to party \( II \) and 2 to party \( III \). Each party has a preference ranking over the candidates. These are shown in the following table.
<table>
<thead>
<tr>
<th>party I (4 voters)</th>
<th>party II (3 voters)</th>
<th>party III (2 voters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

If each voter votes according to his preference and plurality voting is in use, A wins. Hence party A determines the outcome. With 44% of the seats it has 100% of the voting power. However, if both II and III are aware of the distribution of preferences, they have an incentive to agree on either B or C as the candidate to vote for. Should they vote accordingly, parties II and III would have full control over the outcome with 55% of the votes.

Suppose now that the plurality runoff system would be in use. With sincere voting the above preference rankings would first result in a runoff between A and B whereupon B would beat A on the second round. This is in contrast with the outcome under sincere plurality voting. With strategic voting and complete information, party I would have an incentive to vote for C in the first round, thereby securing the victory of C. This would be a better outcome than B for party I. In any event the power distribution again depends on the procedure used.

The same point can be made by considering yet another procedure, viz. amendment system where alternatives are confronted with each other in pairs according to an agenda. In each comparison the majority loser is eliminated and the winner faces the next one. The winner of the last comparison is the overall winner. In this example, with sincere voting, the winner is C the (Condorcet) winner. This is also the result of sophisticated voting. Hence, the smallest party III seems to have the control over the voting outcome under amendment system. This simple example illustrates that different voting procedures may result in a variation of the level of control over the outcomes. It cannot be excluded that the groups with the smallest resources have the largest control over outcomes.

## 5 Power and Causality

The elaboration of various power measures and their discussion is meant to increase our understanding of power in collectivities and also to be of help in the design of voting bodies [see e.g. Gambarelli and Uristani 2009]. A relatively new application of these measures results from their formal equivalence with representations of causality in collective decision making. Given this, it seems a short step to equate power and responsibility.¹

The specification of causality in the case of collective decision making with respect to individual agents cannot be derived from the action and

¹This section derives from Holler [2011].
the result as both are determined by the collectivity. They have to be traced back to decision making and, in general, the decision making process. However, collective decision making has a quality that substantially differs from individual decision making. For instance, an agent may support his favored alternative by voting for another alternative or by not voting at all. Nurmi [1999; 2006] contains a collection of such “paradoxes”. These paradoxes tell us that we cannot derive the contribution of an individual to a particular collective action from the individual’s voting behavior. Trivially, a vote is not a contribution, but a decision. Resources such as power, money, etc. are potential contributions and causality might be related with them if a collective action results. If we refer to resources then causality even follows from votes that do not support the collective action. This is reflected by everyday language that simply states that the Parliament has decided when in fact a decision was made by a majority smaller than 100 per cent. But how can we allocate causality if it is not derived from decisions?

Alternatively, we may assume in what follows that the vote (even in committees) is secret and we do not know who voted “yes” or “no”. Moreover, in general, there are more than two alternatives and the fact that a voter votes “yes” for A in a last pairwise voting only means that he/she prefers A to B or does not want to abstain, but this vote does not tell us why and how alternatives C, D, etc. were excluded. Correspondingly, an adequate concept of causality (and responsibility) does not necessarily presuppose a voting result that is known and discloses who said “yes” and who said “no”.

For an illustration, imagine a five-person committee $N = \{1, 2, 3, 4, 5\}$ that makes a choice between the two alternatives $x$ and $y$. The voting rule specifies that $x$ is chosen if either (i) 1 votes for $x$, or (ii) at least three of the players 2−5 vote for $x$. Let’s assume all individuals vote for $x$. What can be said about causality? Clearly, this is a case of over-determination and the allocation of causation is not straightforward. Alternatively, we may assume that all we get to know is that $x$ is decided, but we do not know who voted for or against it. In both cases, we may conclude on causality by looking at all possible winning coalitions. For example, the action of agent 1 is a member of only one minimally sufficient coalition, i.e., decisive set, while the actions of each of the other four members are in three decisive sets each. If we take the membership in decisive sets as a proxy for causation, and standardize such that the shares of causation adds up to one, then vector

$$h^0 = \left( \frac{1}{13}, \frac{3}{13}, \frac{3}{13}, \frac{3}{13}, \frac{3}{13} \right)$$

represents the degrees of causation. Obviously, this measure of causation coincides with the PGI.

Braham and van Hees [2009: 334], who introduced and discussed the above case, conclude that “this is a questionable allocation of causality.” They add that “by focusing on minimally sufficient conditions, the measure
ignores the fact that anything that players 2–5 can do to achieve \( x \), player 1 can do, and in fact more - he can do it alone.” Taken this view into account, \( h^0 \) seems to violate monotonicity. At least at the first glance we share this specification.

Let’s review the above example. Imagine that \( x \) stands for polluting a lake. Now the lake is polluted, and all five members of \( N \) are under suspicion of having contributed to its pollution. Then \( h^0 \) implies that the share of causation for 1 is significantly smaller than the shares of causation of each of the other four members of \( N \). If responsibility and perhaps even punishment follow from causation then the allocation \( h^0 \) seems highly pathological.\(^2\) As a consequence Braham and van Hees propose to apply the weak NESS instead of the strong one, i.e., not to refer to decisive sets, but to consider sufficient sets instead and count how often an element \( i \) of \( N \) is a necessary element of a sufficient set (i.e., a NESS). Taking care of an adequate standardization so that the shares add up to 1, we get the following allocation of causation:

\[
b^0 = \left( \frac{11}{23}, \frac{3}{23}, \frac{3}{23}, \frac{3}{23}, \frac{3}{23} \right)
\]

The result expressed by \( b^0 \) looks much more convincing than the result proposed by \( h^0 \), doesn’t it? Note that the \( b \)-measure and \( h \)-measure correspond to the Banzhaf index and the PGI, respectively, and can be calculated accordingly.

So far the numerical results propose the weak NESS test and thus the application of the normalized Banzhaf index. However, what happened to alternative \( y \)? If \( y \) represents “no pollution” then the set of decisive sets consists of all subsets of \( N \) that are formed of the actions of agent 1 and the actions of two out of agents 2, 3, 4, and 5. Thus, the actions of 1 are members of six decisive sets while the actions of 2, 3, 4, and 5 are members of three decisive sets each. The corresponding shares are given by the vector

\[
h^* = \left( \frac{1}{3}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \right)
\]

Obviously, \( h^* \) looks much more convincing than \( h^0 \) and the critical interpretation of Braham and van Hees does no longer apply: agent 1 cannot bring about \( y \) on its own, but can cooperate with six different pairs of two other agents to achieve this goal.

Note that the actions (votes) bringing about \( x \) represent an improper game - two (disjoint) “winning” subsets can exist at the same time and the situation suffers from potential over-determination - while the determination of \( y \) can be described by a proper game. The potential over-determination is a challenge to a causal interpretation. However, if there are only two alternatives \( x \) and \( y \) then “not \( x \)” necessarily implies \( y \), irrespective of whether

\(^2\)In cases of firms courts applied market shares to allocate responsibility and damage payments [see Rose-Ackerman 1990].
the (social) result is determined by voting or by polluting. The h-values indicate that it seems to matter what issue we analyze and what questions we raise while the Banzhaf index with respect to \( y \) is identical to the one for \( x : b^0 = b^* \). It takes care of all coalitions that potentially may form, while the PGI only looks at minimally winning ones.

6 Indicator or Measure

Different solution concepts can be thought of as results of choosing not only which properties one likes, but also which examples one wishes to avoid [Aumann, 1977: 471]. This statement also applies when we relate power and causality in the case of collective decision making as the above discussion illustrates. Whether we should apply \( h \) and \( b \), or a third alternative, to measure causation seems an open question, and this paper will not give an answer to it. To conclude, the PGI and thus the strong NESS concept may produce results that are counter-intuitive at first glance. However, in some decision situations they seem to reveal more about the power structure and corresponding causality allocation than the Banzhaf index and the corresponding weak NESS concept. However, if we want to relate responsibility to power then the non-monotonicity as indicated by the strong NESS test and the PGI is quite a challenge. This interpretation presupposes that agent 1 indeed is more powerful than agents 2, 3, 4, and 5, although a smaller power weight is assigned to agent 1 in the game \( v^0 \).

If the collective choice is made through voting then the PGI and the Deegan-Packel index do not guarantee that a voter with a larger share of votes has at least as much responsibility for the collectively determined outcome as a voter with a smaller share. The notorious example is the voting game \( v'' = (51; 35, 20, 15, 15) \). The corresponding set of minimal winning coalitions is \( M(v'') = \{\{1, 2\}, \{1, 3, 4\}, \{1, 3, 5\}, \{1, 4, 5\}, \{2, 3, 4, 5\}\} \). Thus the corresponding PGI is

\[
h'' = \left(\frac{4}{15}, \frac{2}{15}, \frac{3}{15}, \frac{3}{15}, \frac{3}{15}\right).
\]

Obviously, the game \( v'' \) violates LM. However, note that the game \( v'' \) is not decisive: a tie may result and then there is neither winning nor a losing coalition. The Shapley-Shubik index, the Banzhaf index, and the PHI satisfy LM. As discussed in Holler and Napel [2004a, 2004b], it is an open question whether local monotonicity is an axiom or just a property. Aumann [1977: 471] observes:

an important fiction of axiomatics relates to ‘counter-intuitive examples’, in which a solution concept yields outcomes that seem
bizarre. Most axioms appearing in the axiomatizations so seem reasonable on the face of it, and many of them are in fact quite compelling. The fact that a relatively small selection of such axioms is often categoric (determines a unique solution concept), and that different such selections yield different answers, implies that all together, these reasonable-sounding axioms are contradictory.

From the discussion of the 5-player game in the previous section we can learn that nonmonotonicity might indicate that we asked the wrong question: Is the responsibility with respect to keeping the lake clean or is it with polluting the lake? Both alternatives may imply the sharing of the costs of cleaning it. Of course, there is no quantitative answer to this question, but the quantification by the index showed us that there might be a problem with the specification of the game model.

A possible answer of whether the PGI represents a pathology, might be found in this quality-quantity duality: the use of quantity measures to indicate qualitative properties of (voting) games. Whether a game is improper or non-decisive is not a matter of degree. Indicators show read lights or make strange noises when an event happens that has some meaning in a particular context. This does not necessarily mean that the corresponding indicator functions as a measure, but often it does and when it does it summarizes the measured values in the form of signals. What is a relevant and an appropriate signal of course depends on the context and the recipient. Red lights are not very helpful for blind people. What are the relevant and appropriate signals that correspond to power measures? What are the problems that should be uncovered and perhaps even be solved? What are the properties a power measure has to satisfy when it should serve as a signal? These are questions that we cannot answer in a systematic way without reference to a particular issue.

References


Holler, Manfred J. and Napel, Stefan [2004a] Monotonicity of power and power measures, Theory and Decision 56, 93-111.

Holler, Manfred J. and Napel, Stefan [2004b] Local monotonicity of power: Axiom or just a property, Quality and Quantity 38, 637-647.


Nurmi, Hannu [1999] Voting paradoxes and how to deal with them, Springer.


